13. DNTI AND THE 2GFT:

Double Null Triple Injection and the Two General Feedback Theorem

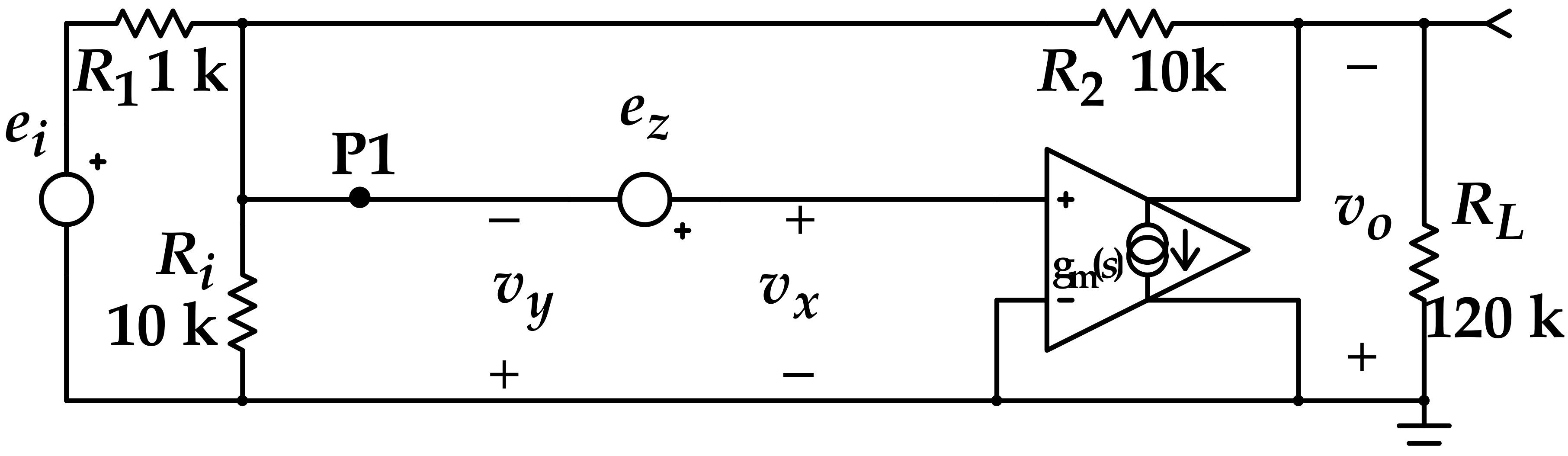
Why two injected test signals are needed in the general case

The GFT with a single injected test signal, as a special case of the Dissection Theorem, incorporates all nonidealities, but still requires an ideal test signal injection point.

Why does a nonideal test signal injection point give "wrong" answers?

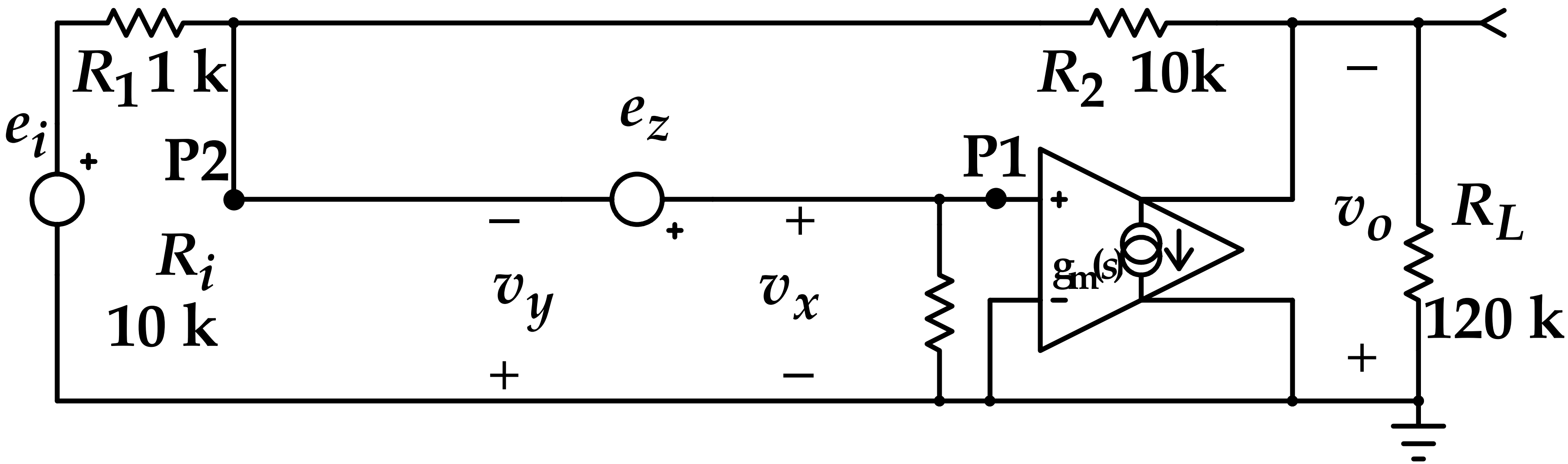
Consider the example circuit already discussed:

Ideal voltage injection point

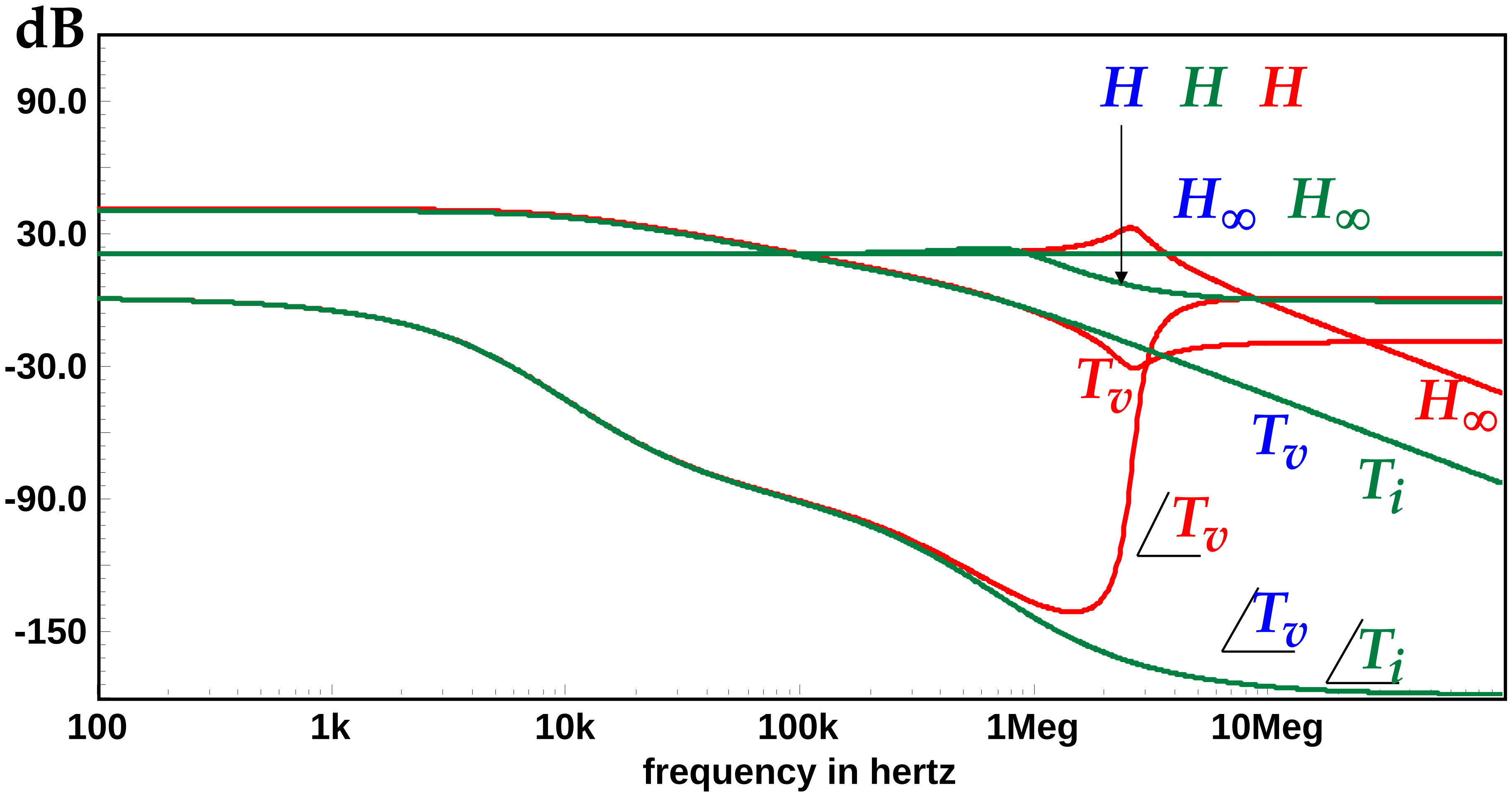


An ideal voltage injection point is where v_y comes from an ideal (zero impedance) voltage generator,
or **where v_x looks into an infinite impedance.**

Test voltage injection on the other side of R_i :

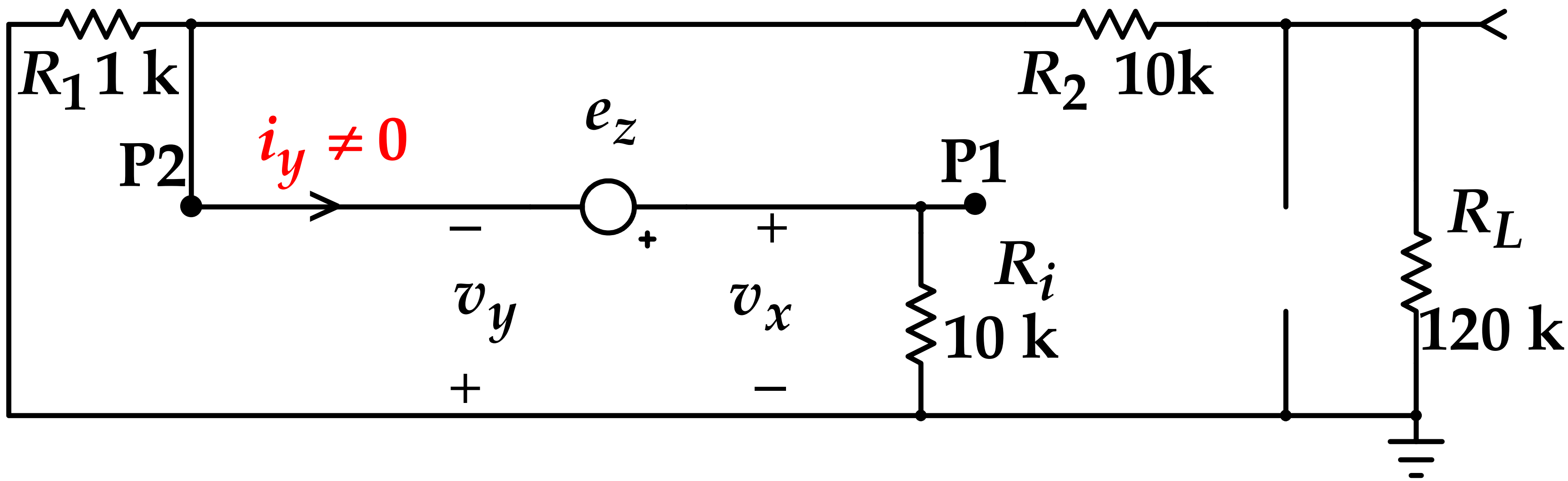


This is a nonideal voltage injection point, and the results for T_v are quite different:



dB
90.0
30.0
-30.0
-90.0
-150
100
1k
10k
100k
1Meg
10Meg
frequency in hertz
H
H
H
H_∞
H_∞
T_v
H_∞
T_v
T_i
$\angle T_v$
$\angle T_v$
$\angle T_i$

Reason for the difference in T_v:



Consider $T_v(\omega \to \infty)$: opamp gain goes to zero

$$v_y = [R_1 \| (R_2 + R_L)] i_y \approx R_1 i_y$$

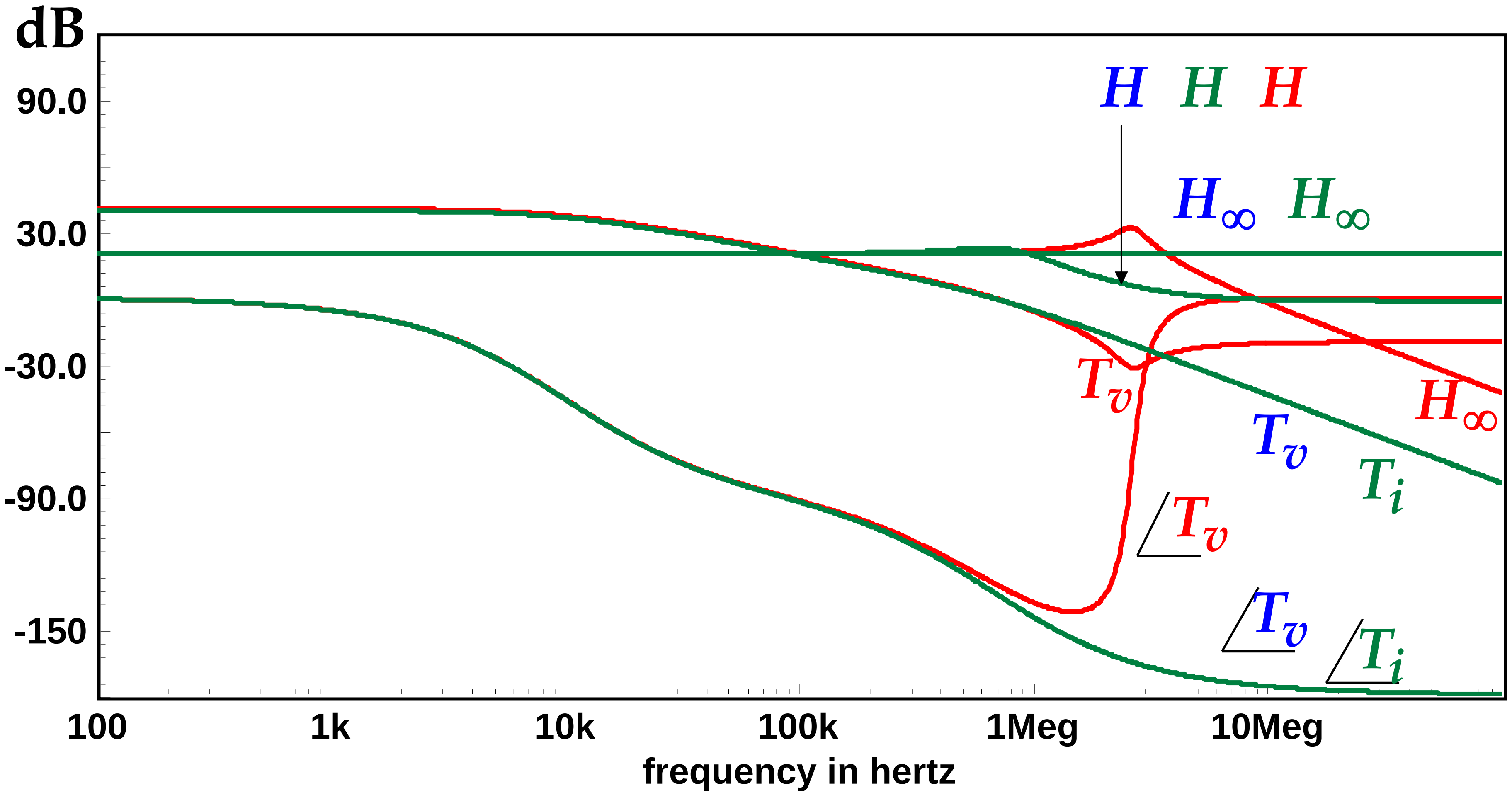
$$v_x = R_i i_y$$

so $$T_v = \frac{v_y}{v_x} = \frac{R_1}{R_i} \neq 0$$

A nonideal injection point gives a u_y / u_x that depends not only on T, but also on the impedance ratio (looking forward and backward) from the injection point.

A nonideal injection point gives a u_y / u_x that depends not only on T, but also on the impedance ratio (looking forward and backward) from the injection point.

A nonideal injection point also gives a H^{u_y} that is different from the desired H_∞ :



dB
90.0
30.0
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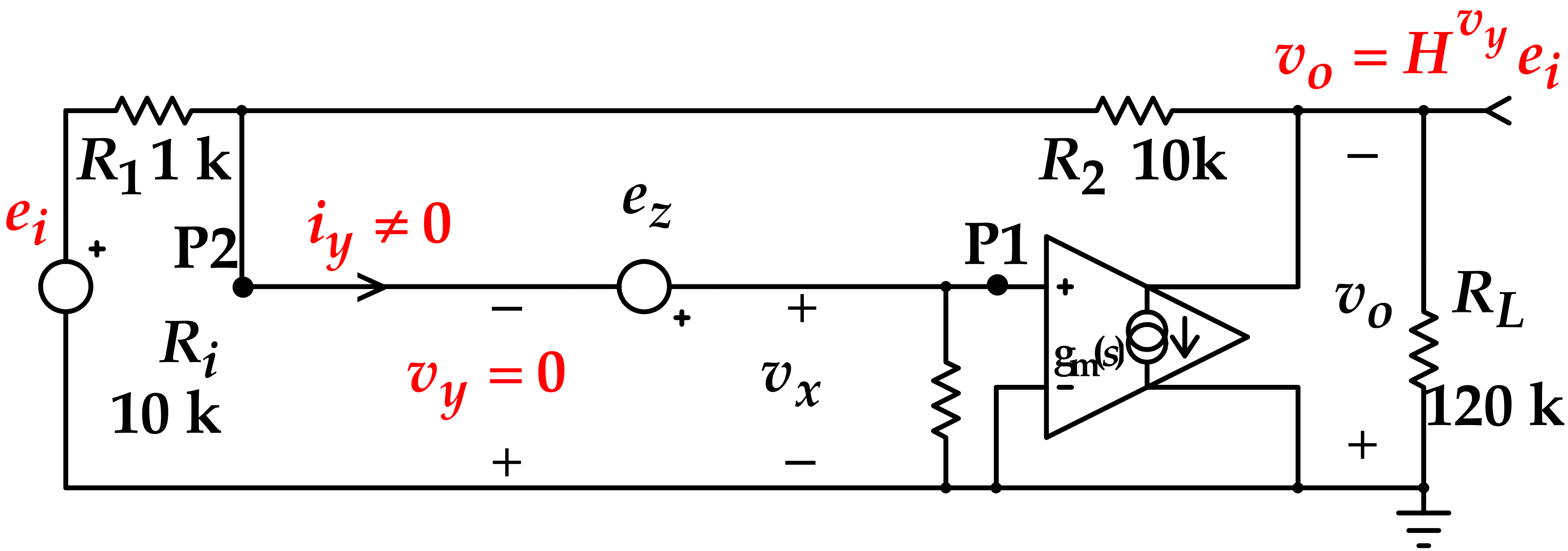
-90.0
-150
100
1k
10k
100k
1Meg
10Meg
frequency in hertz
H
H
H
H∞
H∞
H∞
Tv
Tv
Ti
∠Tv
∠Tv
∠Ti

Reason for the difference in H^{v_y}:



Again, $i_y \neq 0$ **so** $H^{v_y} = \left.\dfrac{v_o}{e_i}\right|_{v_y=0} \neq \dfrac{R_2}{R_1}$,

but depends also on the opamp gain.

An extension of the Basic Feedback Theorem is needed:

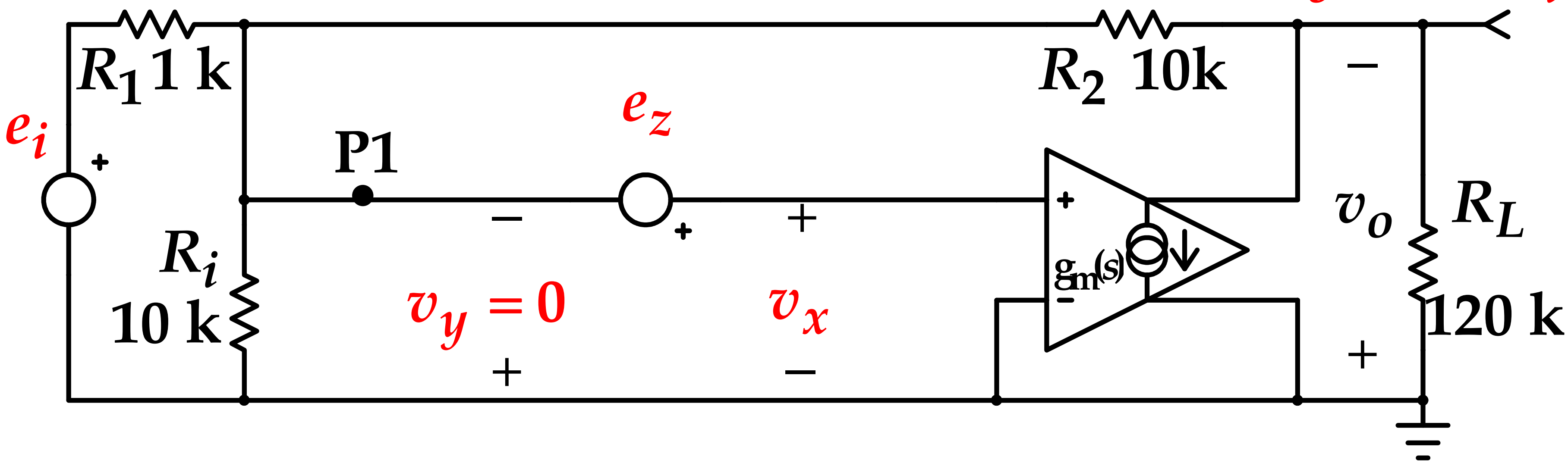
To set up $H_\infty = 1/K$, the reciprocal of the feedback ratio, both the error voltage v_y and the error current i_y must be nulled.

In the preceding example with injected test voltage e_z, the desired result $H_\infty = 1/K$ was obtained because nulled error voltage automatically implied nulled error current:

$$H_\infty = \left.\frac{v_o}{e_i}\right|_{v_y=0} \equiv H^{v_y}$$

A superscript indicates the signal being nulled

$v_o = H^{v_y} e_i$

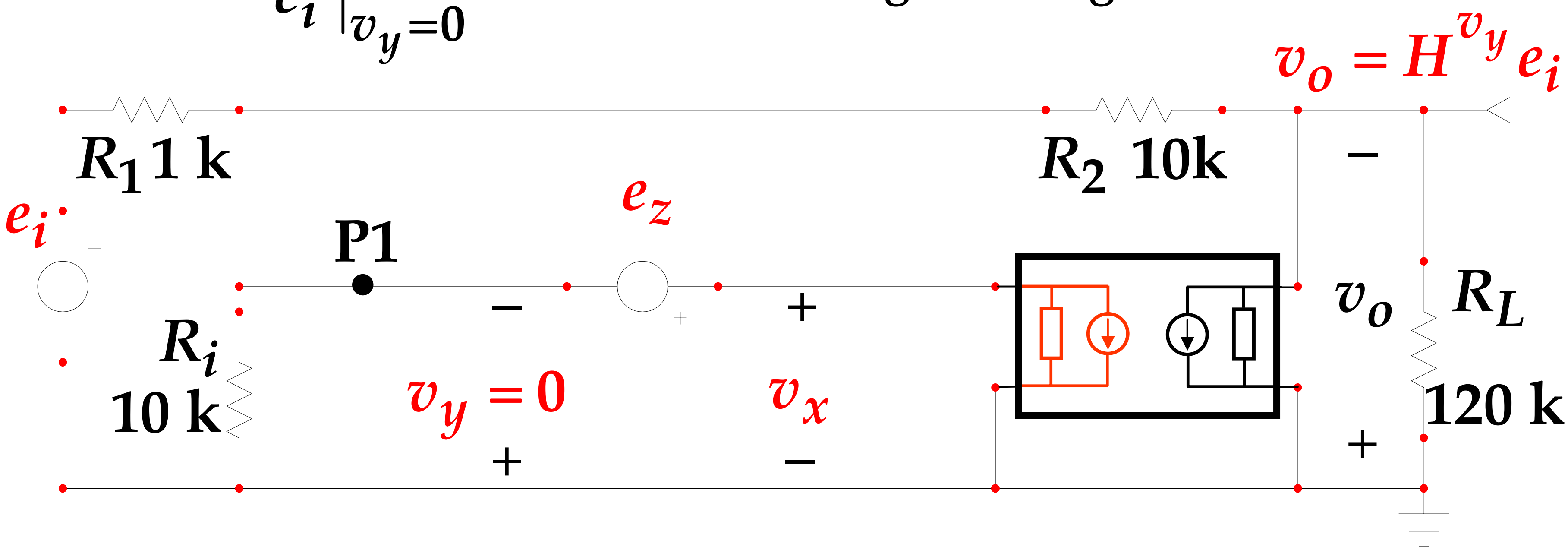


$$H^{v_y} = \frac{R_2}{R_1} = 10 \Rightarrow 20 \text{ dB}$$

In a realistic circuit model, the injection point is not ideal and the X terminal looks into a 2-port model that exhibits noninfinite input impedance and nonzero reverse transmission:

$$H_\infty = \left.\frac{v_o}{e_i}\right|_{v_y=0} \equiv H^{v_y}$$

A superscript indicates the signal being nulled



When v_y is nulled, the error current is *not* nulled, and H_∞ is not equal to R_2 / R_1.

A nonideal voltage injection point implies that when v_y is nulled, i_y is not nulled.

A nonideal current injection point implies that when i_y is nulled, v_y is not nulled.

The 2GFT: The Final Solution

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_\infty D D_n$$

To make H_∞ equal $1/K$:

What is needed is a way to null simultaneously both the error voltage v_y and the error current i_y.

The answer is obvious:

Since

One signal can be nulled by mutual adjustment of two independent sources (ndi, null double injection)

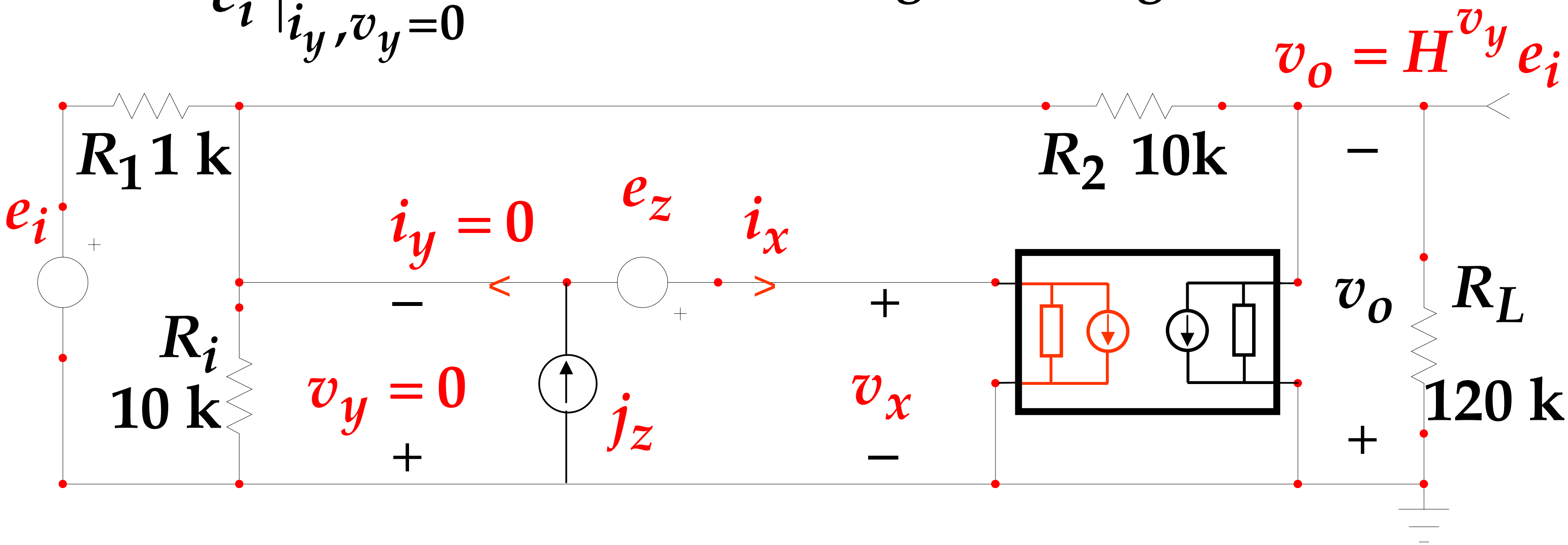
Two signals can be nulled by mutual adjustment of three independent sources (dnti, double null triple injection).

Therefore:

In the presence of the input signal e_i, inject both voltage and current test signals so that both the error voltage v_y and the error current i_y can be nulled.

$$H_\infty = \left.\frac{v_o}{e_i}\right|_{i_y, v_y = 0} \equiv H^{i_y v_y}$$

Superscripts indicate the signals being nulled



When v_y and i_y are both nulled, H_∞ is again equal to R_2 / R_1, the reciprocal of the feedback ratio K.

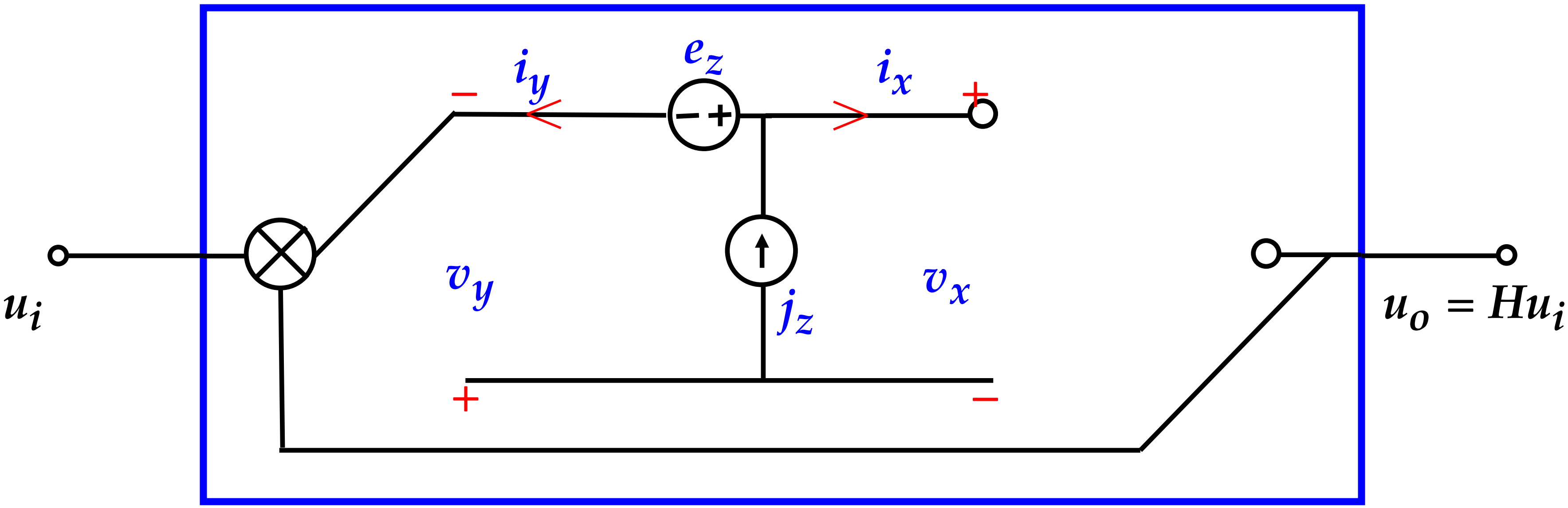
The format of the GFT remains the same:

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}}$$

where now $H_\infty = H^{i_y v_y}$**, and** T **and** T_n **contain both voltage and current loop gains.**

Since the GFT and the EET are special cases of the Dissection Theorem for a single injected test signal, and the 2EET is a special case of the 2DT, it is to be expected that the 2GFT for two injected test signals would have the same format as the 2EET with impedance ratios replaced by return ratios.

The 2GFT tells how to find T when both current and voltage loop gains are present.

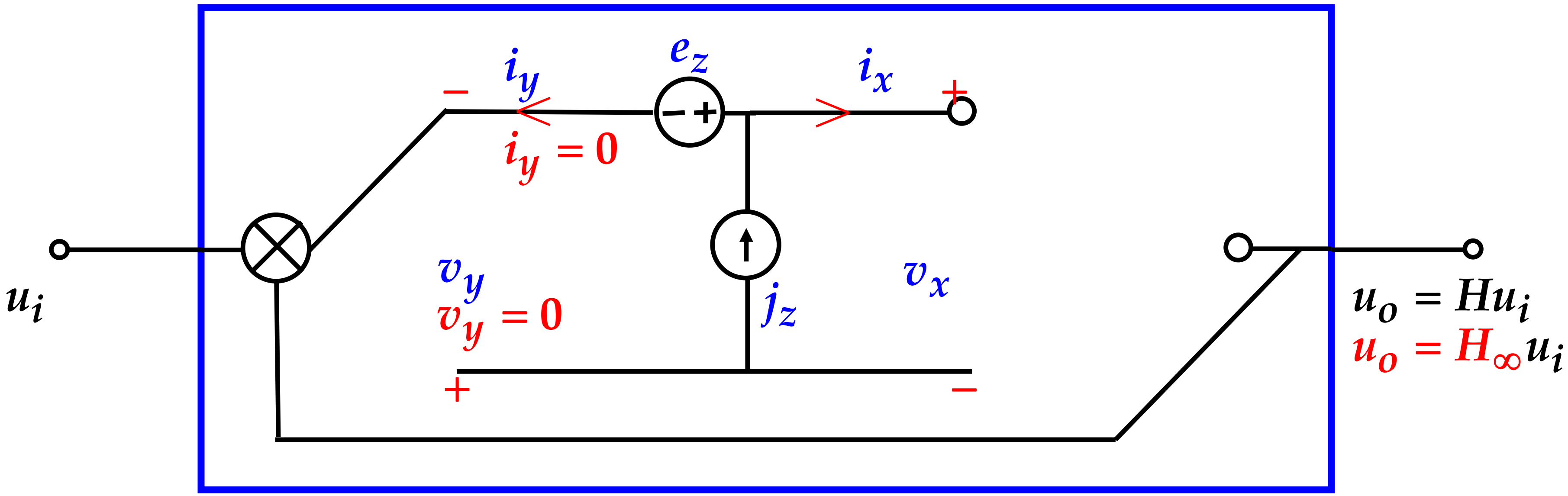


$$H = H^{i_y v_y} \frac{1 + \frac{1}{T_{ni}^{v_y}} + \frac{1}{T_{nv}^{i_y}} + K_n \frac{1}{T_{ni}^{v_y}} \frac{1}{T_{nv}^{i_y}}}{1 + \frac{1}{T_{i}^{v_y}} + \frac{1}{T_{v}^{i_y}} + K_d \frac{1}{T_{i}^{v_y}} \frac{1}{T_{v}^{i_y}}}$$

where K_d and K_n are interaction parameters:

$$K_d \equiv \frac{T_i^{v_y}}{T_i^{v_x}} = \frac{T_v^{i_y}}{T_v^{i_x}} \qquad K_n \equiv \frac{T_{ni}^{v_y}}{T_{ni}^{v_x}} = \frac{T_{nv}^{i_y}}{T_{nv}^{i_x}}$$

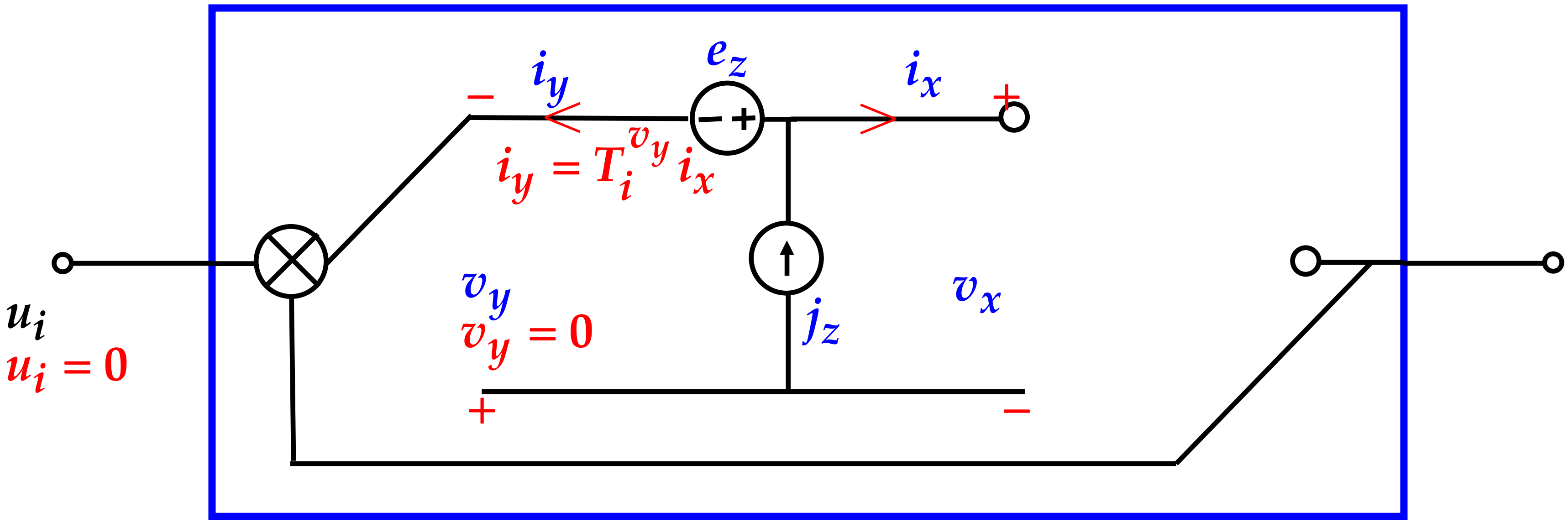
Interpretations:



$$H_\infty \equiv H^{i_y v_y} \equiv \left.\frac{u_o}{u_i}\right|_{i_y, v_y = 0}$$ **= ideal closed-loop gain**

$H^{i_y v_y}$ **is a double null triple injection (dnti) calculation, which is even simpler and shorter than an ndi calculation.**

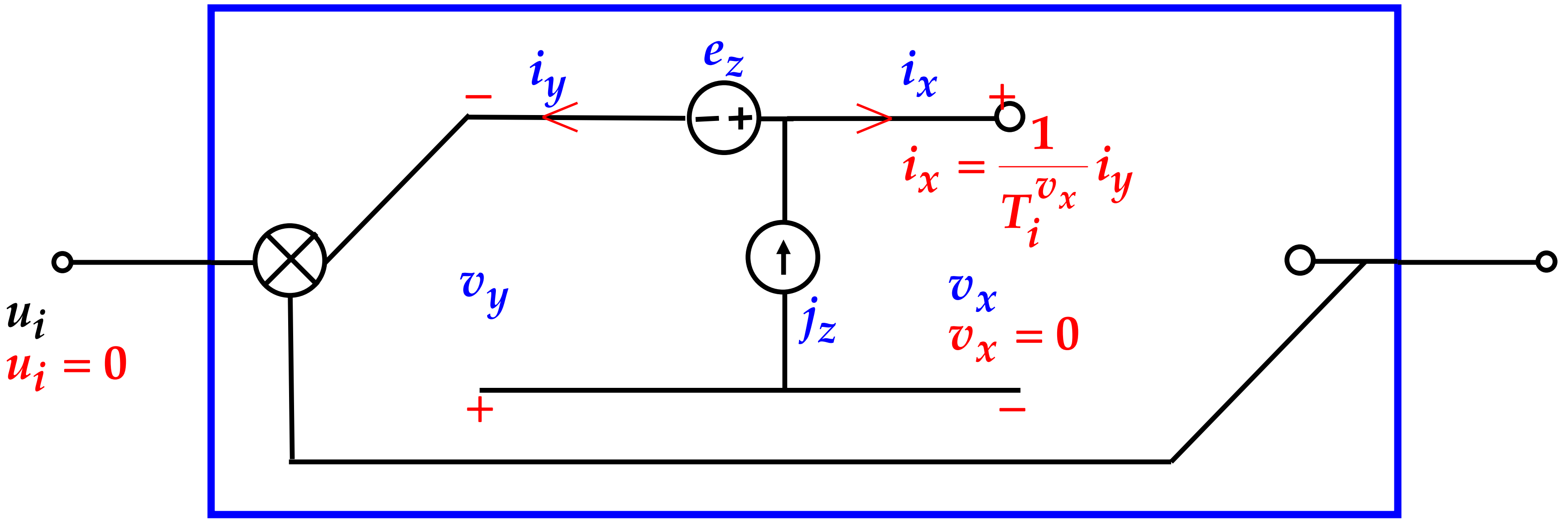
Interpretations:



$$T_i^{v_y} \equiv \left.\frac{i_y}{i_x}\right|_{v_y=0}$$ **= short-circuit current loop gain**

$T_i^{v_y}$ **is a null double injection (ndi) calculation, which is simpler and shorter than an si calculation.**

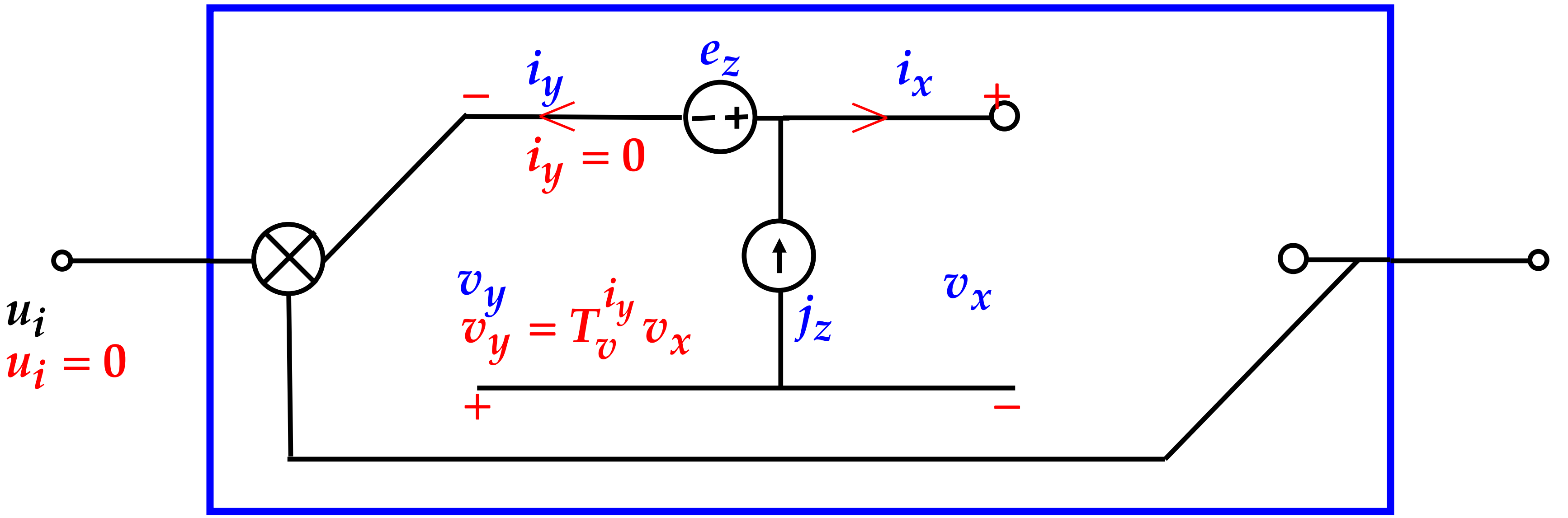
Interpretations:



$$\frac{1}{T_i^{v_x}} \equiv \left.\frac{i_x}{i_y}\right|_{v_x=0}$$ = short-circuit **reverse** current loop gain

$T_i^{v_x}$ is a null double injection (ndi) calculation.

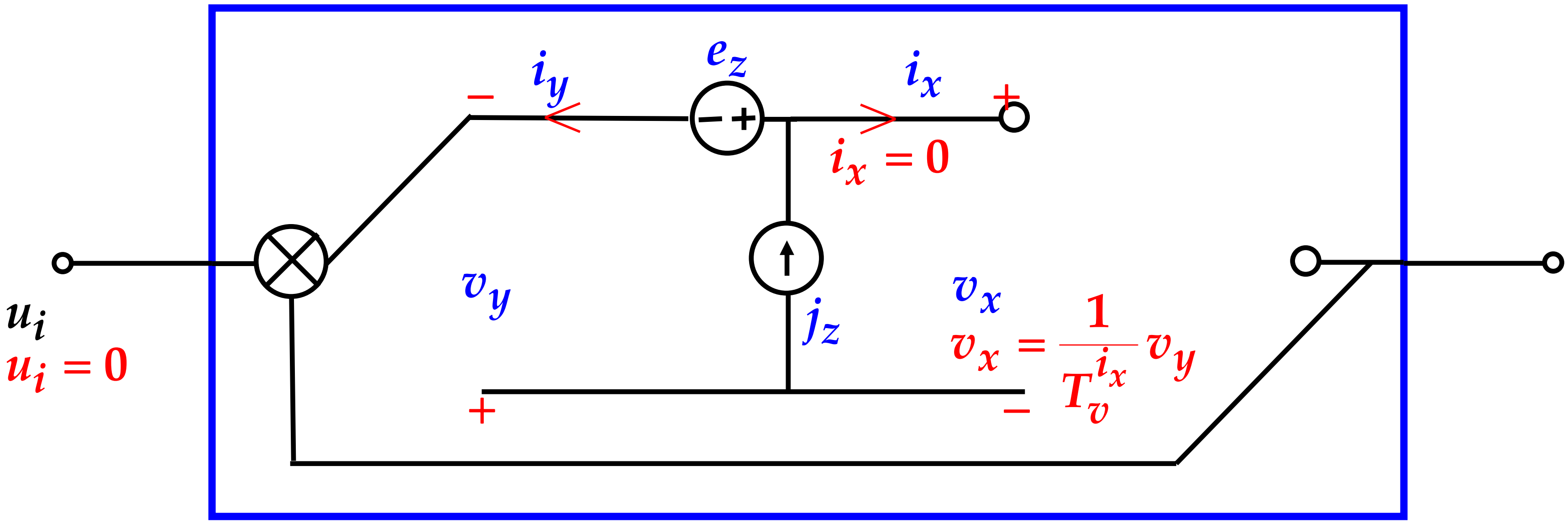
Interpretations:



$$T_v^{i_y} \equiv \left.\frac{v_y}{v_x}\right|_{i_y=0}$$ **= open-circuit voltage loop gain**

$T_v^{i_y}$ **is a null double injection (ndi) calculation.**

Interpretations:



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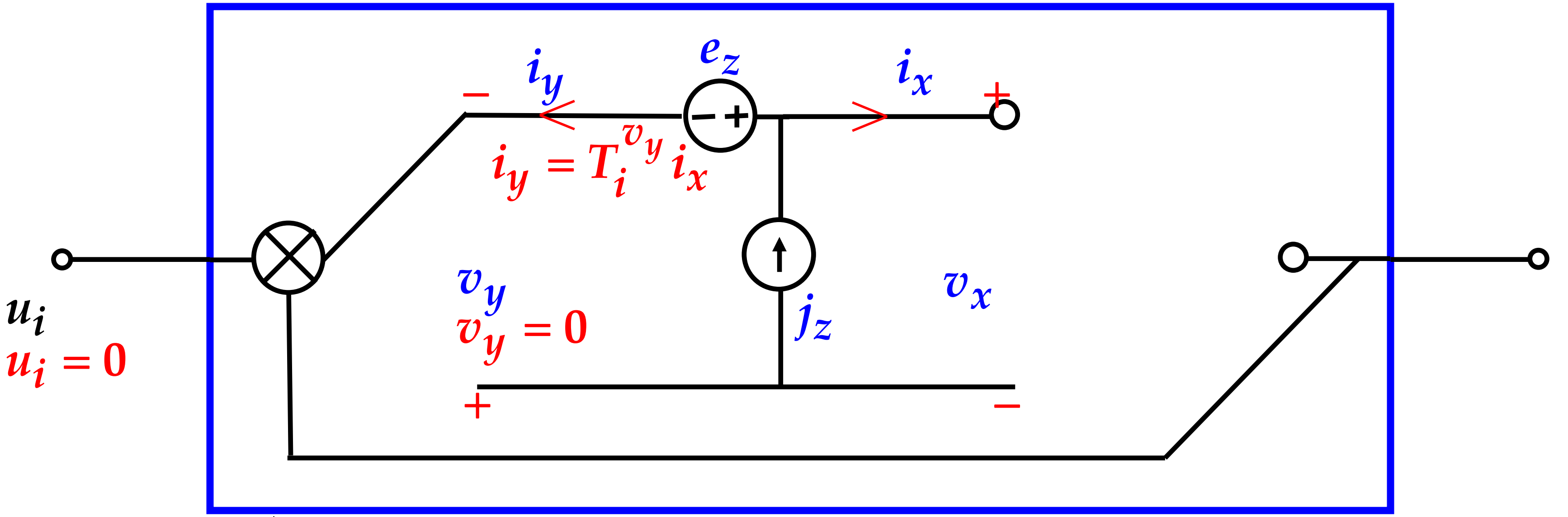

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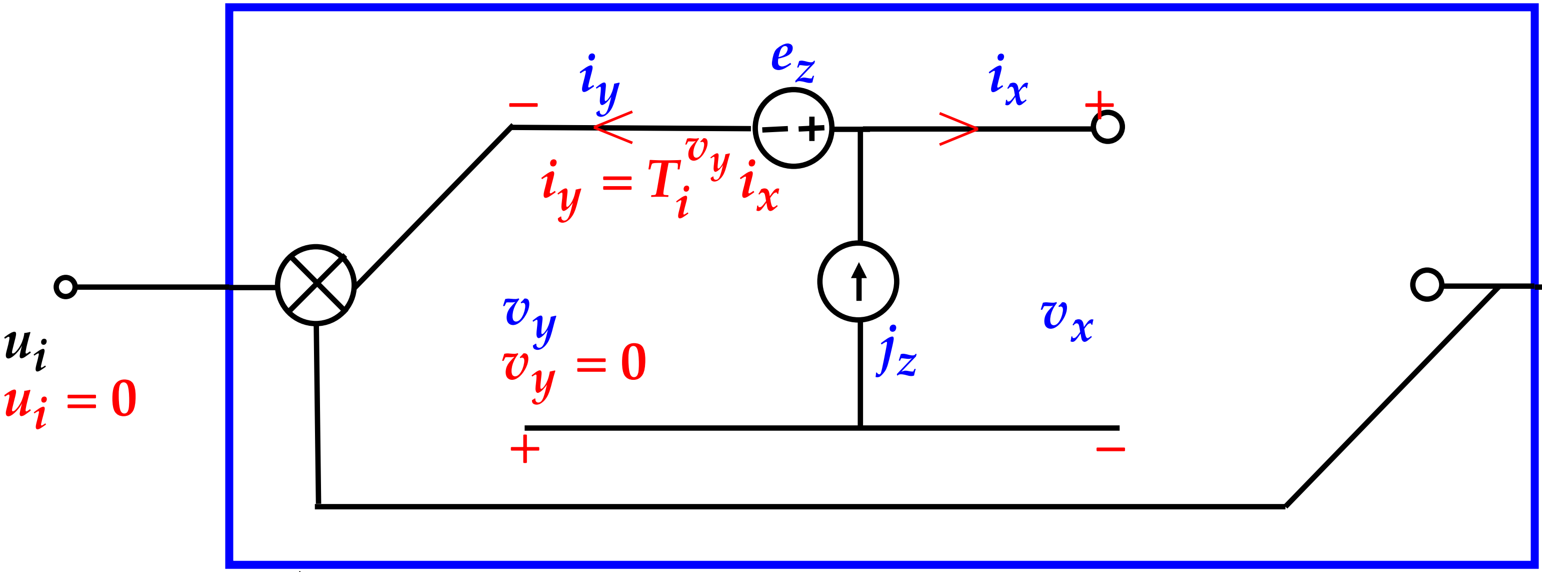
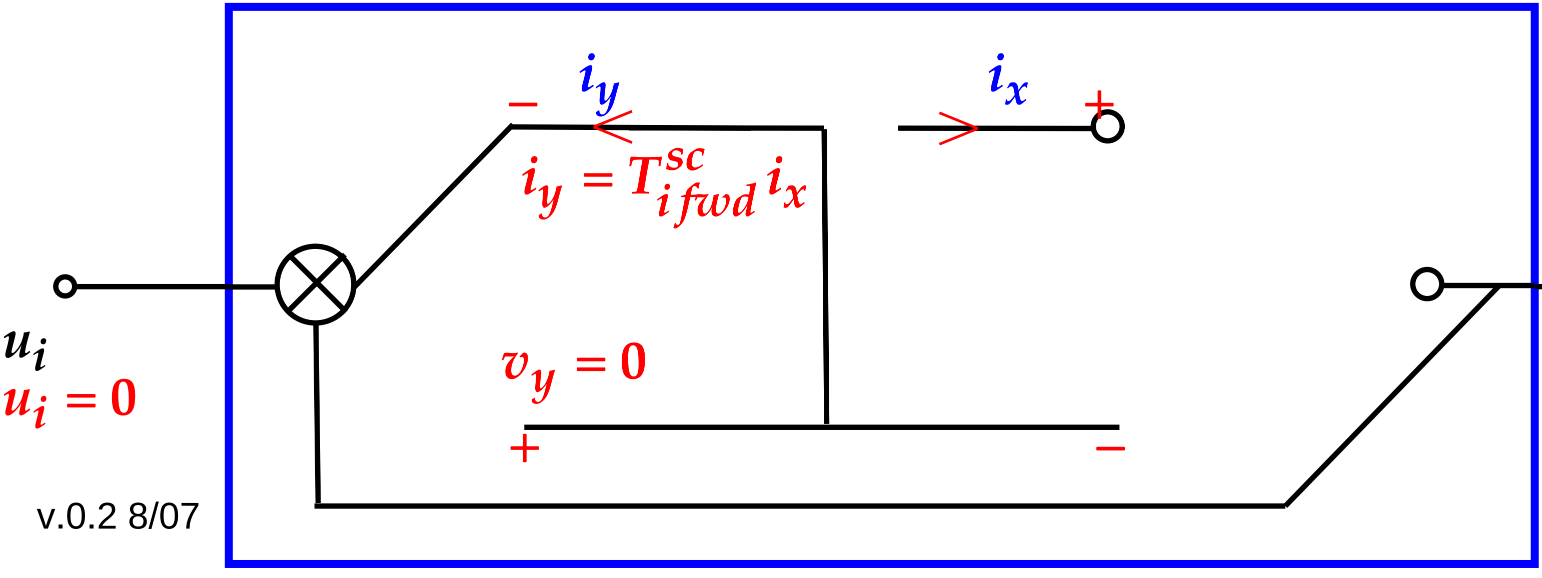
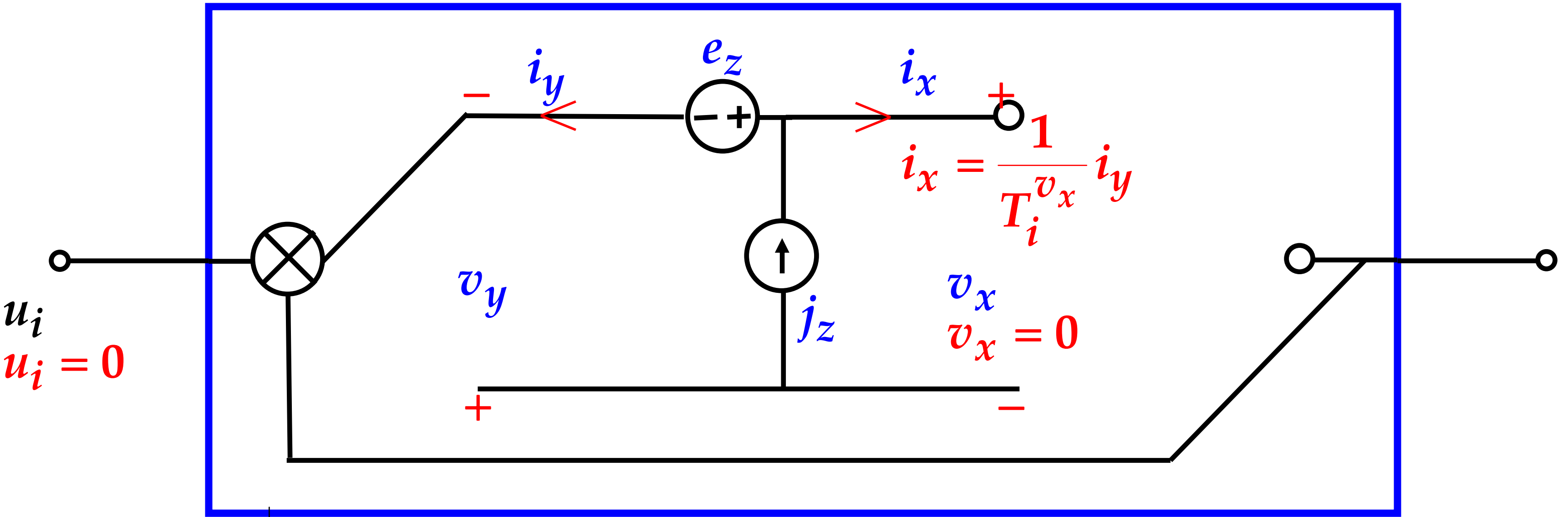


$$T_i^{v_y} \equiv \left.\frac{i_y}{i_x}\right|_{v_y=0}$$ = **short-circuit forward current loop gain** $\equiv T_{i\,fwd}^{sc}$

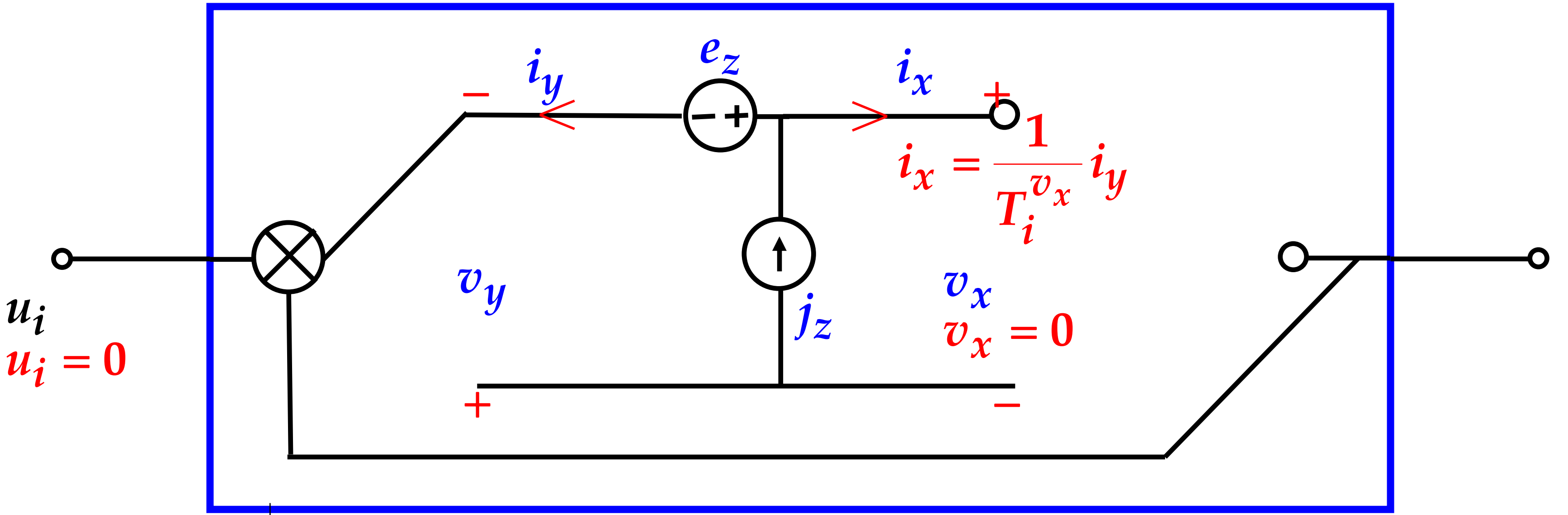


Definitions:

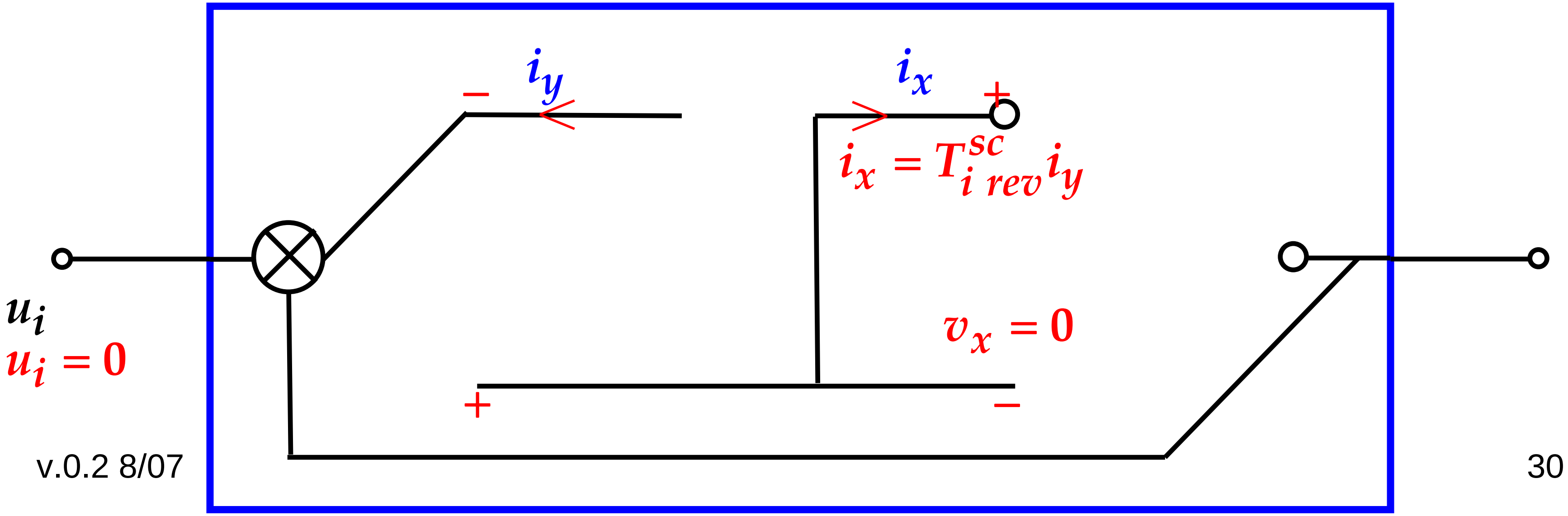


$$\frac{1}{T_i^{v_x}} \equiv \left.\frac{i_x}{i_y}\right|_{v_x=0}$$ = **short-circuit reverse current loop gain**

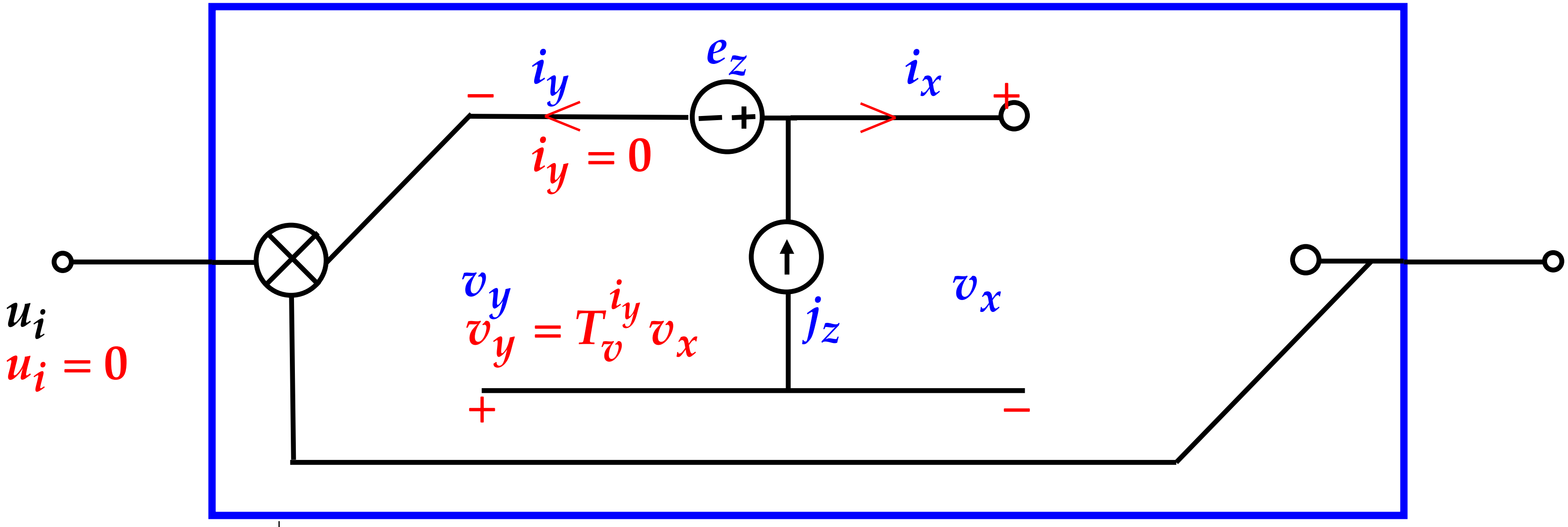
Definitions:



$$\frac{1}{T_i^{v_x}} \equiv \left.\frac{i_x}{i_y}\right|_{v_x=0}$$ = short-circuit reverse current loop gain $\equiv T_{i\ rev}^{sc}$

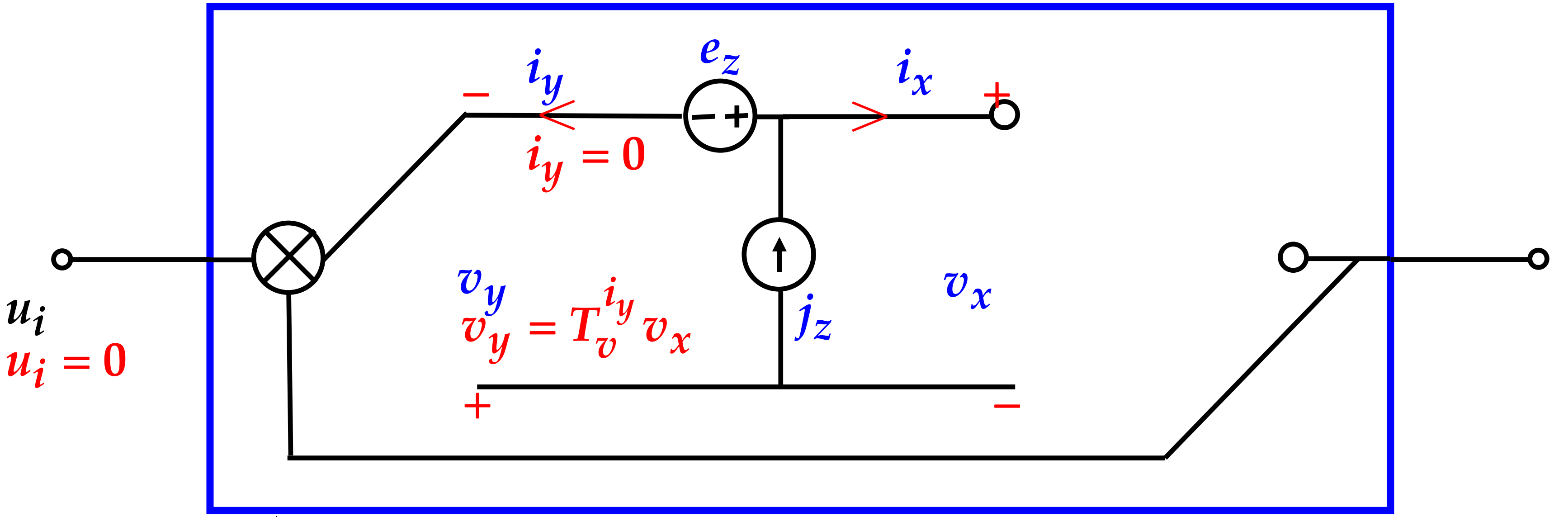


Definitions:

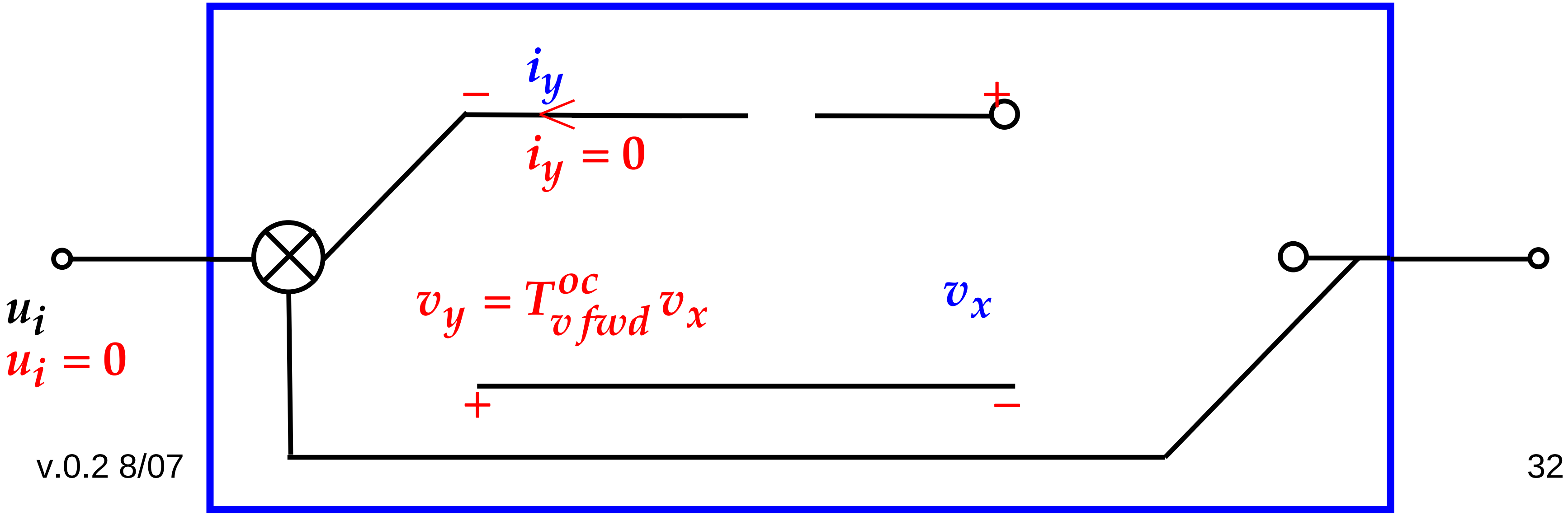


$$T_v^{i_y} \equiv \left. \frac{v_y}{v_x} \right|_{i_y=0}$$ = **open-circuit forward voltage loop gain**

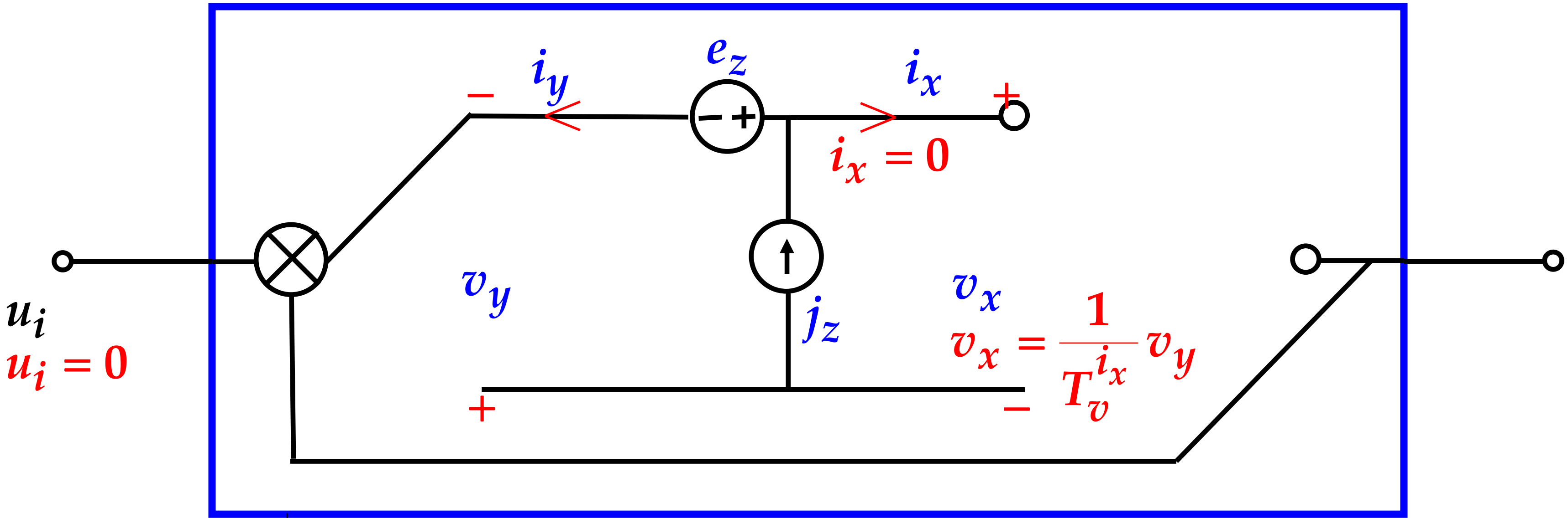
Definitions:



$$T_v^{i_y} \equiv \left.\frac{v_y}{v_x}\right|_{i_y=0}$$ = open-circuit **forward** voltage loop gain $\equiv T_{v\,fwd}^{oc}$

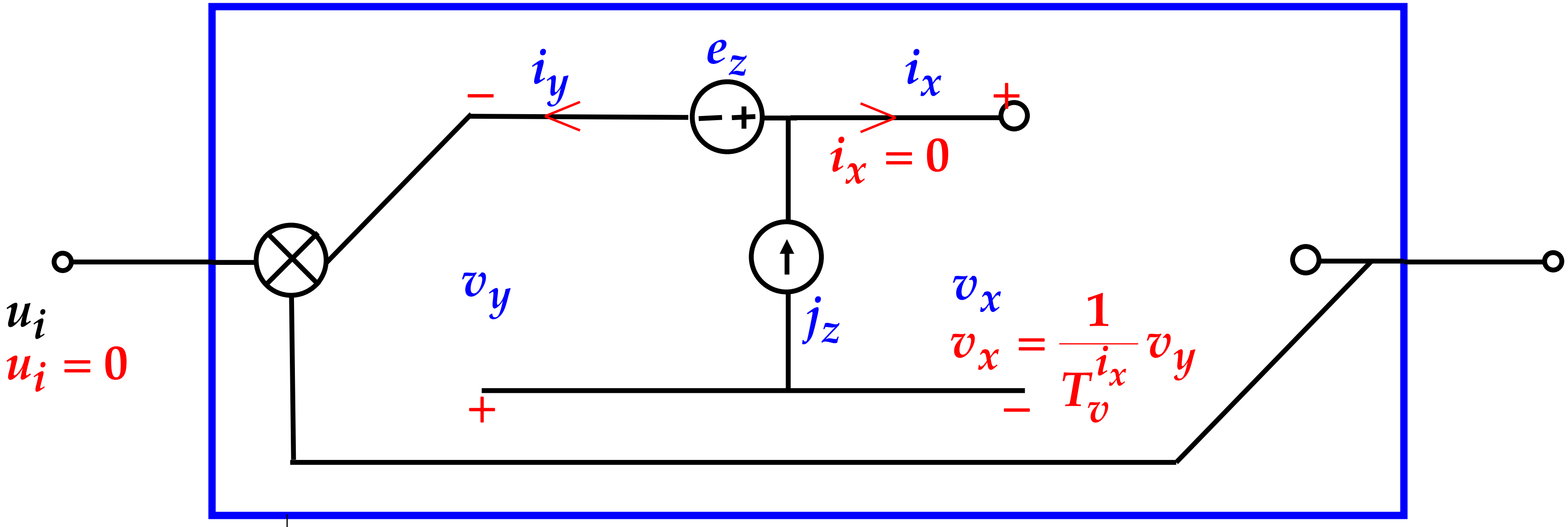


Definitions:

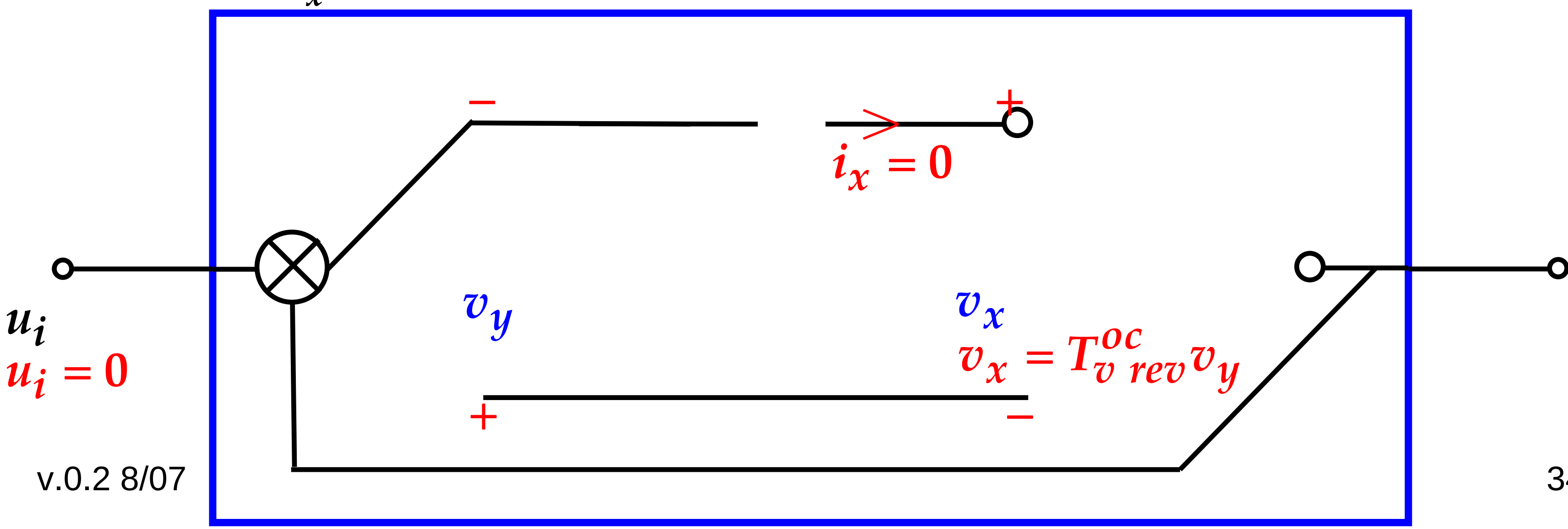


$$\frac{1}{T_v^{i_x}} \equiv \left. \frac{v_x}{v_y} \right|_{i_x=0}$$ = open-circuit reverse voltage loop gain

Definitions:



$$\frac{1}{T_v^{i_x}} \equiv \left.\frac{v_x}{v_y}\right|_{i_x=0}$$ = open-circuit **reverse** voltage loop gain $\equiv T_{v\ rev}^{oc}$



Summary so far:

The 2GFT: $$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}}$$

$$\frac{1}{T} \equiv \frac{1}{T_i^{v_y}} + \frac{1}{T_v^{i_y}} + K_d \frac{1}{T_i^{v_y}} \frac{1}{T_v^{i_y}}$$

in which the second level TF T, the total loop gain, can be written in terms of the newly defined third level TFs:

$$T_i^{v_y} \equiv \left.\frac{i_y}{i_x}\right|_{v_y=0}$$ = **short-circuit forward current loop gain** $\equiv T_{i\,fwd}^{sc}$

$$T_v^{i_y} \equiv \left.\frac{v_y}{v_x}\right|_{i_y=0}$$ = **open-circuit forward voltage loop gain** $\equiv T_{v\,fwd}^{oc}$

$$\frac{1}{T_i^{v_x}} \equiv \left.\frac{i_x}{i_y}\right|_{v_x=0}$$ = **short-circuit reverse current loop gain** $\equiv T_{i\,rev}^{sc}$

$$\frac{1}{T_v^{i_x}} \equiv \left.\frac{v_x}{v_y}\right|_{i_x=0}$$ = **open-circuit reverse voltage loop gain** $\equiv T_{v\,rev}^{oc}$

$$\frac{1}{T} \equiv \frac{1}{T_{i\,fwd}^{sc}} + \frac{1}{T_{v\,fwd}^{oc}} + K_d \frac{1}{T_{i\,fwd}^{sc}} \frac{1}{T_{v\,fwd}^{oc}}$$

and the redundant interaction parameter K_d is

$$K_d \equiv \frac{T_i^{v_y}}{T_i^{v_x}} = \frac{T_v^{i_y}}{T_v^{i_x}} \qquad \textbf{or} \qquad K_d \equiv T_{i\,fwd}^{sc} T_{i\,rev}^{sc} = T_{v\,fwd}^{oc} T_{v\,rev}^{oc}$$

In words, the redundancy relation says that

K_d = **product of short-circuit forward and reverse current loop gains** = **product of open-circuit forward and reverse voltage loop gains**

The expression for T discloses that nonzero reverse loop gain enters only through K_d.

In the absence of reverse loop gain, T reduces to the forward loop gain T_{fwd} given by

$$\frac{1}{T_{fwd}} = \frac{1}{T^{sc}_{i\,fwd}} + \frac{1}{T^{oc}_{v\,fwd}} \qquad \textbf{or} \qquad T_{fwd} = T^{sc}_{i\,fwd} \,\Big\|\, T^{oc}_{v\,fwd}$$

In words, T_{fwd} is the parallel combination of the short-circuit forward current gain and the open circuit forward voltage gain, and is therefore dominated by whichever one is smaller.

Now that reverse current and voltage loop gains have been identified, a reverse loop gain T_{rev} can be defined:

$$\frac{1}{T_{rev}} = \frac{1}{T_{i\ rev}^{sc}} + \frac{1}{T_{v\ rev}^{oc}} \qquad \text{or} \qquad T_{rev} = T_{i\ rev}^{sc} \| T_{v\ rev}^{oc}$$

From the redundancy relation $K_d \equiv T_{i\,fwd}^{sc} T_{i\ rev}^{sc} = T_{v\,fwd}^{oc} T_{v\ rev}^{oc}$,

T_{rev} **can be written in terms of** K_d **as**

$$T_{rev} = \frac{K_d}{T_{i\,fwd}^{sc} + T_{v\,fwd}^{oc}}$$

or

$$K_d \frac{1}{T_{i\,fwd}^{sc}} \frac{1}{T_{v\,fwd}^{oc}} = T_{rev}\left(\frac{1}{T_{i\,fwd}^{sc}} + \frac{1}{T_{v\,fwd}^{oc}}\right)$$

Finally, by substitution for K_d in terms of T_{rev}, the total loop gain T

$$\frac{1}{T} \equiv \frac{1}{T_{i\,fwd}^{sc}} + \frac{1}{T_{v\,fwd}^{oc}} + K_d \frac{1}{T_{i\,fwd}^{sc}} \frac{1}{T_{v\,fwd}^{oc}}$$

becomes

$$\frac{1}{T} = \left(\frac{1}{T_{i\,fwd}^{sc}} + \frac{1}{T_{v\,fwd}^{oc}} \right) + T_{rev} \left(\frac{1}{T_{i\,fwd}^{sc}} + \frac{1}{T_{v\,fwd}^{oc}} \right) = \frac{1 + T_{rev}}{T_{fwd}}$$

or

$$\frac{1}{T} = \frac{1 + T_{rev}}{T_{fwd}}$$

or

$$T = \frac{T_{fwd}}{1 + T_{rev}}$$

A 1975 paper* showed how the loop gain could be found by combining the current and voltage loop gains obtained by simultaneous injection of current and voltage test signals at a nonideal injection point. That result is actually $T_{fwd} = T_{i\,fwd}^{sc} \,\Big\|\, T_{v\,fwd}^{oc}$ obtained above.

However, the 1975 paper was based on a model that excluded nonzero reverse loop gain, and so the exact 2GFT result can be considered an extension of the previous limited result.

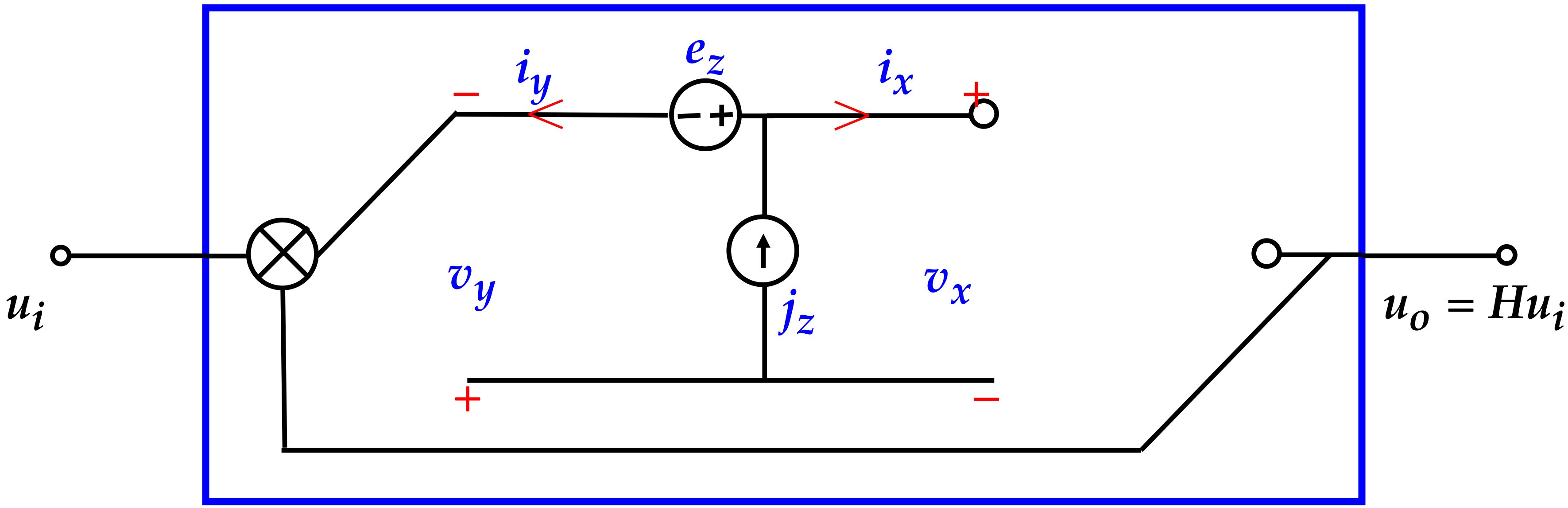
In summary, the total loop gain T given by the 2GFT is

$$T = \frac{T_{fwd}}{1 + T_{rev}}$$

where $T_{fwd} = T_{i\,fwd}^{sc} \,\Big\|\, T_{v\,fwd}^{oc}$ and $T_{rev} = T_{i\,rev}^{sc} \,\Big\|\, T_{v\,rev}^{oc}$

* R.D.Middlebrook, "Measurement of loop gain in feedback systems," Int. J. Electronics, 1975, vol. 38, No. 4, pp. 485 – 512.

The 2GFT also tells how to find T_n when both current and voltage loop gains are present.

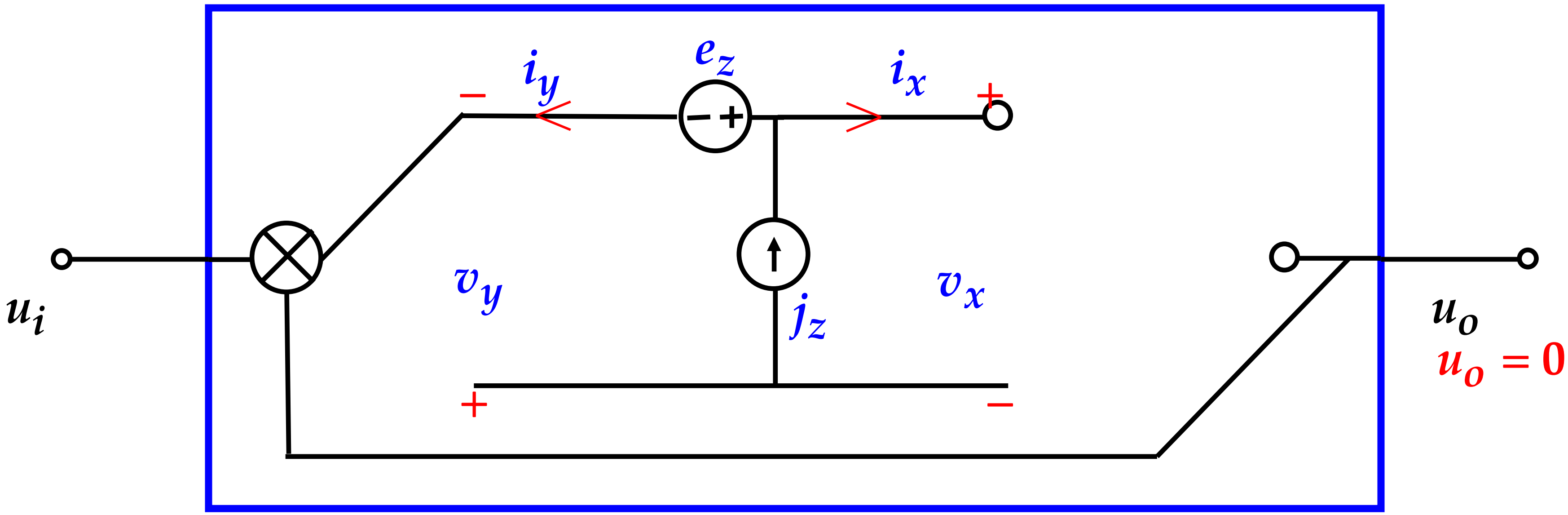


$$H = H^{i_y v_y} \frac{1 + \dfrac{1}{T_{ni}^{v_y}} + \dfrac{1}{T_{nv}^{i_y}} + K_n \dfrac{1}{T_{ni}^{v_y}} \dfrac{1}{T_{nv}^{i_y}}}{1 + \dfrac{1}{T_i^{v_y}} + \dfrac{1}{T_v^{i_y}} + K_d \dfrac{1}{T_i^{v_y}} \dfrac{1}{T_v^{i_y}}}$$

where K_d and K_n are interaction parameters:

$$K_d \equiv \frac{T_i^{v_y}}{T_i^{v_x}} = \frac{T_v^{i_y}}{T_v^{i_x}} \qquad K_n \equiv \frac{T_{ni}^{v_y}}{T_{ni}^{v_x}} = \frac{T_{nv}^{i_y}}{T_{nv}^{i_x}}$$

The four T_n's are defined in the same way as the four T's, except that the input u_i is restored and the output u_o is also nulled.



The four T_n's are double null triple injection (dnti) calculations, as is $H^{i_y v_y}$, which are even simpler and shorter than ndi calculations.

Since the structure of the null loop gain T_n is exactly the same as that of the loop gain T, the discussion for T can simply be repeated for T_n with the extra modifier "null", as follows:

Discussion of the total null loop gain T_n :

The 2GFT: $$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}}$$

$$\frac{1}{T_n} = \frac{1}{T^{sc}_{ni\,fwd}} + \frac{1}{T^{oc}_{nv\,fwd}} + K_n \frac{1}{T^{sc}_{ni\,fwd} T^{oc}_{nv\,fwd}}$$

in which the second level TF T_n, the total null loop gain, can be written in terms of the newly defined third level TFs:

$$T^{v_y}_{ni} \equiv \left.\frac{i_y}{i_x}\right|_{u_o, v_y = 0}$$ = **short-circuit forward null current loop gain** $\equiv T^{sc}_{ni\,fwd}$

$$T^{i_y}_{nv} \equiv \left.\frac{v_y}{v_x}\right|_{u_o, i_y = 0}$$ = **open-circuit forward null voltage loop gain** $\equiv T^{oc}_{nv\,fwd}$

$$\frac{1}{T^{v_x}_{ni}} \equiv \left.\frac{i_x}{i_y}\right|_{u_o, v_x = 0}$$ = **short-circuit reverse null current loop gain** $\equiv T^{sc}_{ni\,rev}$

$$\frac{1}{T^{i_x}_{nv}} \equiv \left.\frac{v_x}{v_y}\right|_{u_o, i_x = 0}$$ = **open-circuit reverse null voltage loop gain** $\equiv T^{oc}_{nv\,rev}$

$$\frac{1}{T_n} = \frac{1}{T_{ni\,fwd}^{sc}} + \frac{1}{T_{nv\,fwd}^{oc}} + K_n \frac{1}{T_{ni\,fwd}^{sc} T_{nv\,fwd}^{oc}}$$

and the redundant interaction parameter K_n is

$$K_n \equiv \frac{T_{ni}^{v_y}}{T_{ni}^{v_x}} = \frac{T_{nv}^{i_y}}{T_{nv}^{i_x}} \qquad \text{or} \qquad K_n = T_{ni\,fwd}^{sc} T_{ni\,rev}^{sc} = T_{nv\,fwd}^{oc} T_{nv\,rev}^{oc}$$

In words, the redundancy relation says that

K_n = **product of short-circuit forward and reverse null current loop gains** = **product of open-circuit forward and reverse null voltage loop gains**

The expression for T_n discloses that nonzero reverse null loop gain enters only through K_n.

In the absence of reverse null loop gain, T_n reduces to the forward null loop gain $T_{n\,fwd}$ given by

$$\frac{1}{T_{n\,fwd}} = \frac{1}{T^{sc}_{ni\,fwd}} + \frac{1}{T^{oc}_{nv\,fwd}} \quad \text{or} \quad T_{n\,fwd} = T^{sc}_{ni\,fwd} \,\|\, T^{oc}_{nv\,fwd}$$

In words, $T_{n\,fwd}$ is the parallel combination of the short-circuit forward null current loop gain and the open circuit forward null voltage loop gain, and is therefore dominated by whichever one is smaller.

Now that reverse current and voltage null loop gains have been identified, a reverse null loop gain $T_{n\,rev}$ can be defined:

$$\frac{1}{T_{n\,rev}} = \frac{1}{T_{ni\,rev}^{sc}} + \frac{1}{T_{nv\,rev}^{oc}} \qquad \textbf{or} \qquad T_{n\,rev} = T_{ni\,rev}^{sc} \| T_{nv\,rev}^{oc}$$

From the redundancy relation $K_n = T_{ni\,fwd}^{sc} T_{ni\,rev}^{sc} = T_{nv\,fwd}^{oc} T_{nv\,rev}^{oc}$,

$T_{n\,rev}$ **can be written in terms of** K_n **as**

$$T_{n\,rev} = \frac{K_n}{T_{ni\,fwd}^{sc} + T_{nv\,fwd}^{oc}}$$

or

$$K_n \frac{1}{T_{ni\,fwd}^{sc}} \frac{1}{T_{nv\,fwd}^{oc}} = T_{n\,rev} \left(\frac{1}{T_{ni\,fwd}^{sc}} + \frac{1}{T_{nv\,fwd}^{oc}} \right)$$

Finally, by substitution for K_n in terms of $T_{n\,rev}$, the total null loop gain T_n

$$\frac{1}{T_n} = \frac{1}{T^{sc}_{ni\,fwd}} + \frac{1}{T^{oc}_{nv\,fwd}} + K_n \frac{1}{T^{sc}_{ni\,fwd} T^{oc}_{nv\,fwd}}$$

becomes

$$\frac{1}{T_n} = \left(\frac{1}{T^{sc}_{ni\,fwd}} + \frac{1}{T^{oc}_{nv\,fwd}} \right) + T_{n\,rev} \left(\frac{1}{T^{sc}_{ni\,fwd}} + \frac{1}{T^{oc}_{nv\,fwd}} \right) = \frac{1 + T_{n\,rev}}{T_{n\,fwd}}$$

or

$$\frac{1}{T_n} = \frac{1 + T_{n\,rev}}{T_{n\,fwd}}$$

or

$$T_n = \frac{T_{n\,fwd}}{1 + T_{n\,rev}}$$

In summary, the 2GFT in terms of the newly defined TFs is

$$H = H_{\infty} \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}}$$ **where**

$$H_{\infty} = H^{i_y v_y}$$

$$T = \frac{T_{fwd}}{1+T_{rev}}$$ **in which** $T_{fwd} = T_{i\,fwd}^{sc} \| T_{v\,fwd}^{oc}$ **and** $T_{rev} = T_{i\,rev}^{sc} \| T_{v\,rev}^{oc}$

$$T_n = \frac{T_{n\,fwd}}{1+T_{n\,rev}}$$ **in which** $T_{n\,fwd} = T_{ni\,fwd}^{sc} \| T_{nv\,fwd}^{oc}$ **and** $T_{n\,rev} = T_{ni\,rev}^{sc} \| T_{nv\,rev}^{oc}$

Reminder:

$$H = H_{\infty}\frac{T}{1+T} + H_0\frac{1}{1+T} = H_{\infty}\frac{1+\frac{1}{T_n}}{1+\frac{1}{T}} = H_{\infty}DD_n$$

Redundancy Relation: $\dfrac{H_{\infty}}{H_0} = \dfrac{T_n}{T}$

In order for the *definitions* of the four second-level TFs to have the following *interpretations*

$H_{\infty} =$ **ideal closed loop gain**

$T =$ **principal loop gain**

$H_0 =$ **direct forward transmission**

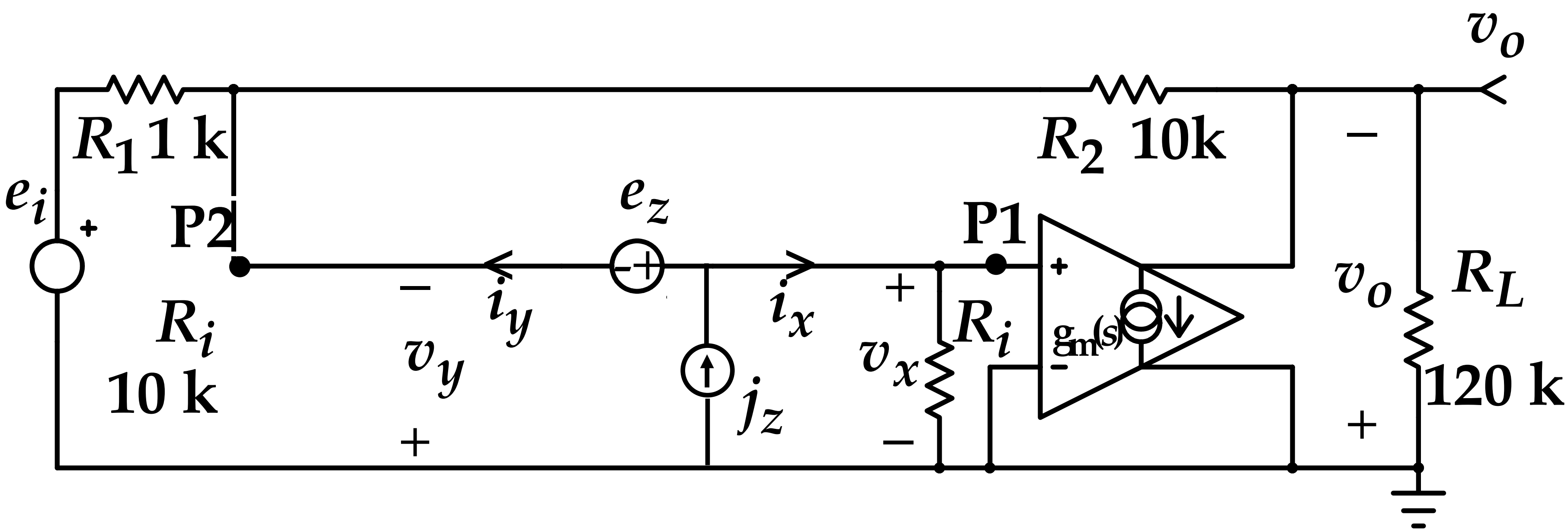
$T_n =$ **null loop gain**

it is necessary that the Test Signal Injection Configuration satisfy two conditions:

1. **The test signal u_z (e_z and/or j_z) must be injected so that u_y (i_y and/or v_y) is the error signal.**

2. **The test signal must be injected inside the major loop, but outside any minor loops.**

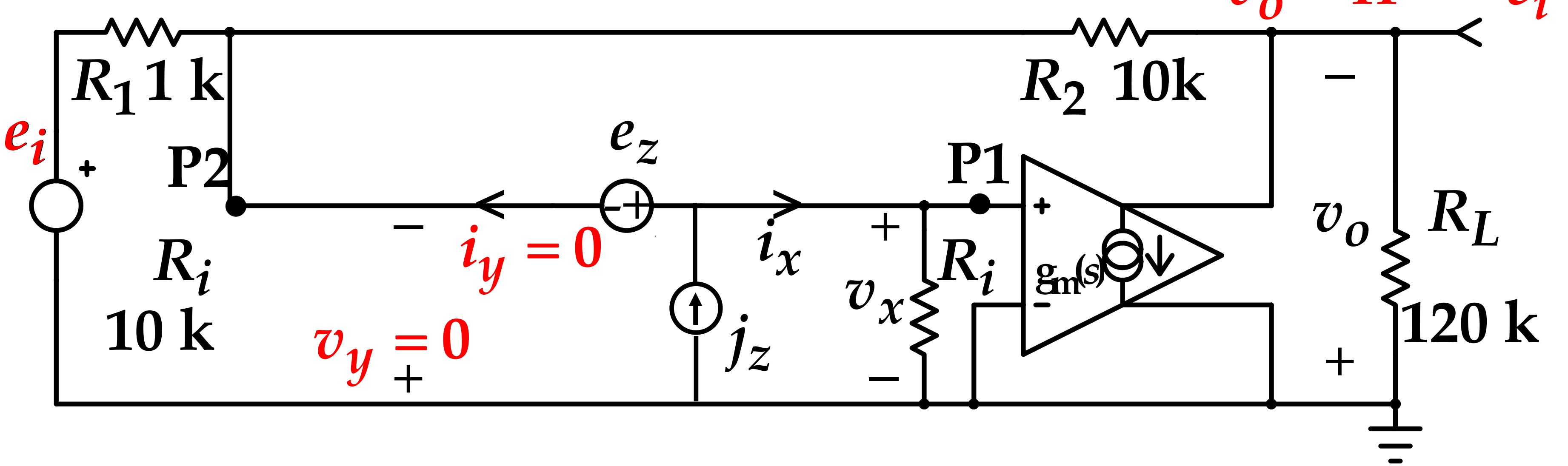
Inverting Opamp with nonideal injection point



Inject both a test voltage and a test current

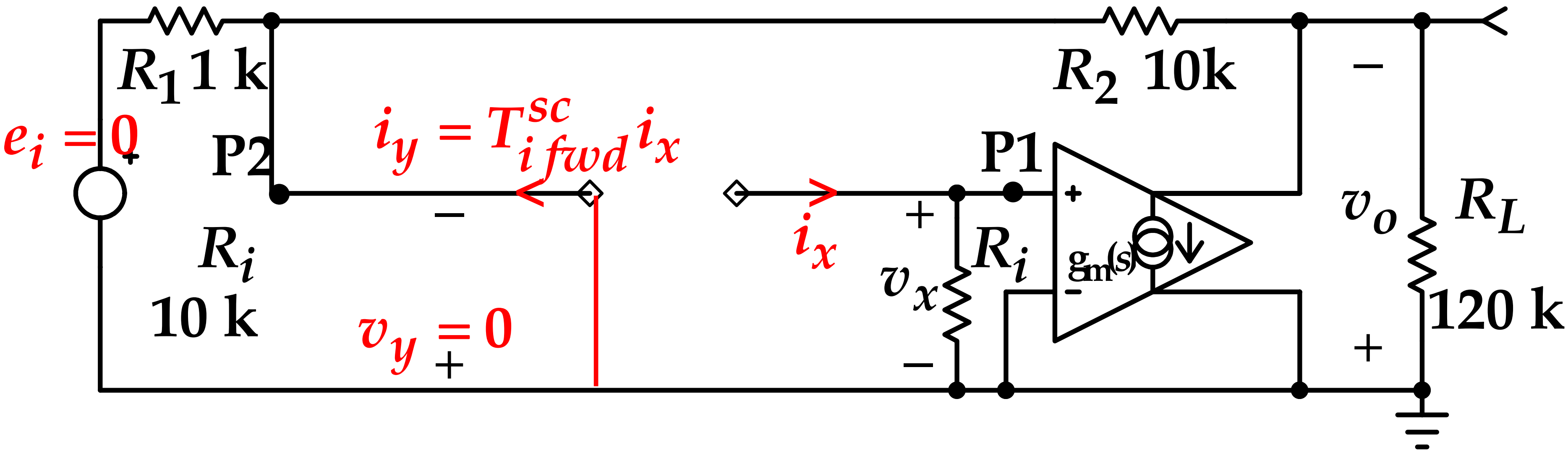
$$H = H_\infty \frac{1 + \frac{1}{T_n}}{1 + \frac{1}{T}}$$

$$H_\infty = \frac{v_o}{e_i}\bigg|_{i_y, v_y = 0} \equiv H^{i_y v_y}$$



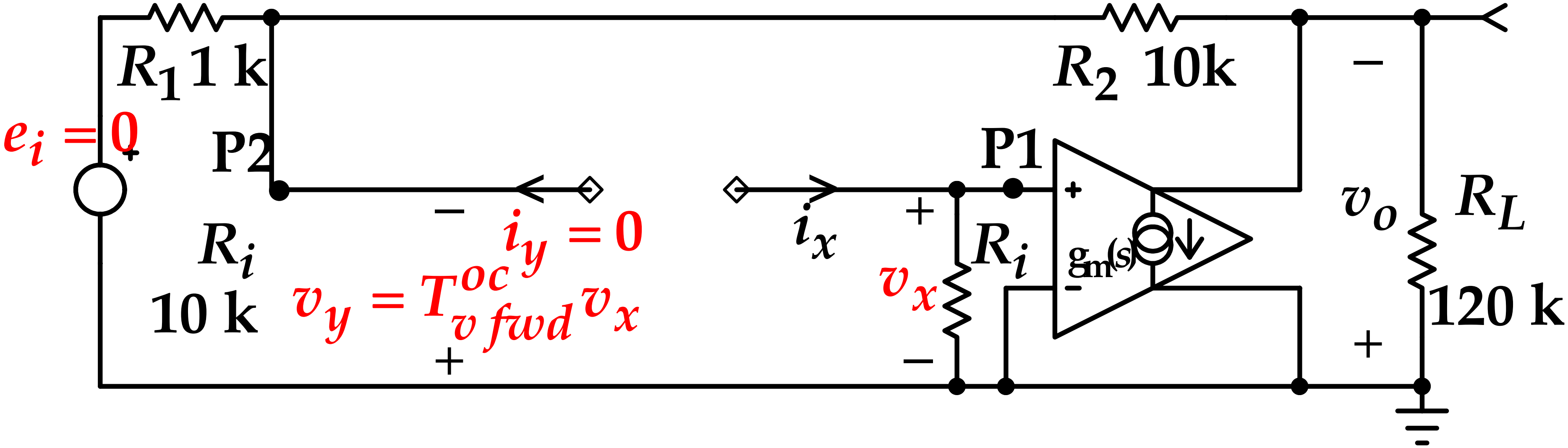
$$H^{i_y v_y} = \frac{R_2}{R_1} = 10 \Rightarrow 20 \text{ dB}$$

$$T_i^{v_y} = \left.\frac{i_y}{i_x}\right|_{e_i, v_y = 0} \equiv T_{i\,fwd}^{sc}$$



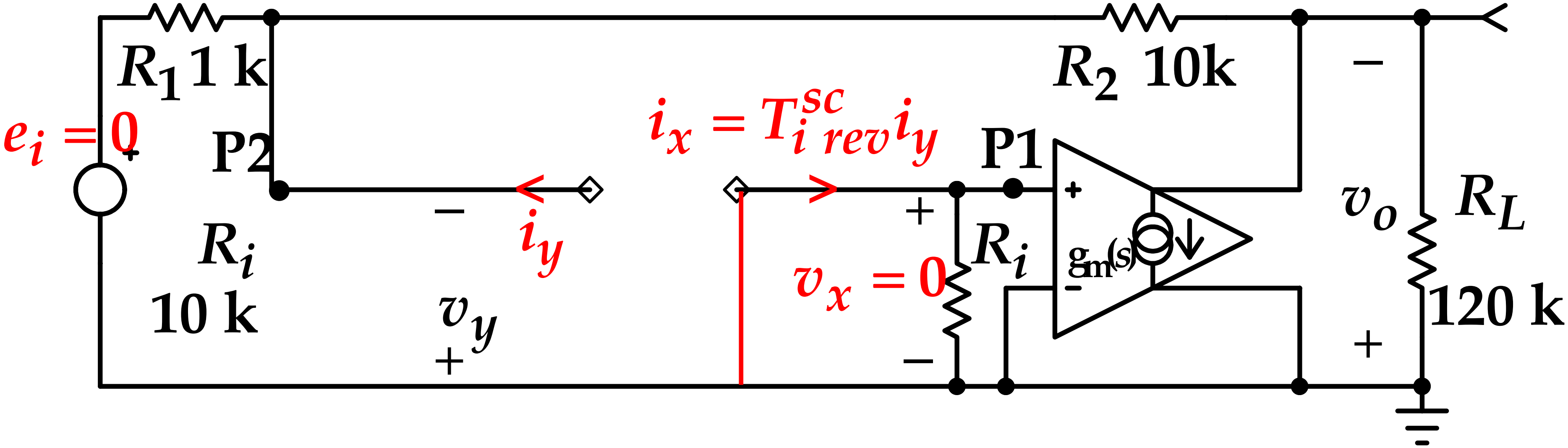
$$T_{i\,fwd}^{sc} = R_i g_m(s) \frac{R_L}{R_L + R_2}$$

$$T_v^{i_y} = \left.\frac{v_y}{v_x}\right|_{e_i, i_y = 0} \equiv T_{v\,fwd}^{oc}$$



$$T_{v\,fwd}^{oc} = g_m(s) \frac{R_L}{R_L + R_2 + R_1} R_1$$

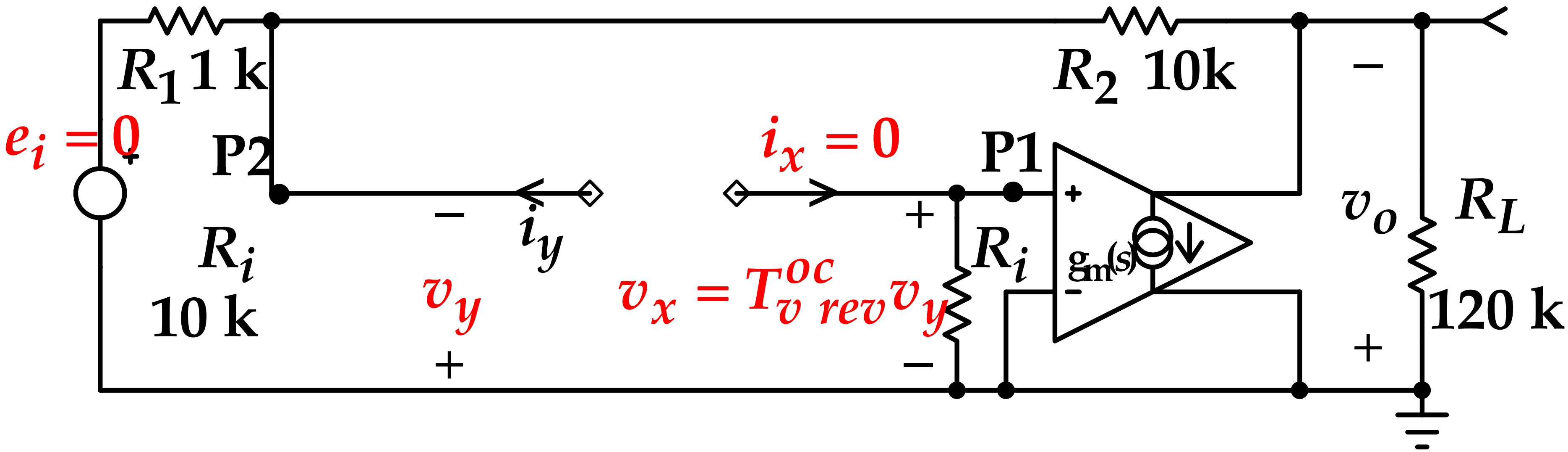
$$\frac{1}{T_i^{v_x}} \equiv \left.\frac{i_x}{i_y}\right|_{v_x=0} \equiv T_{i\,rev}^{sc}$$



$v_x = 0$, and because there is no reverse current loop gain,

$i_x = 0$ so $T_{i\,rev}^{sc} = 0$.

$$\frac{1}{T_v^{i_x}} \equiv \left.\frac{v_x}{v_y}\right|_{i_x=0} \equiv T_{v\ rev}^{oc}$$



$i_x = 0$, and because there is no reverse voltage loop gain,

$v_x = 0$ so $T_{v\ rev}^{oc} = 0$.

Finally,

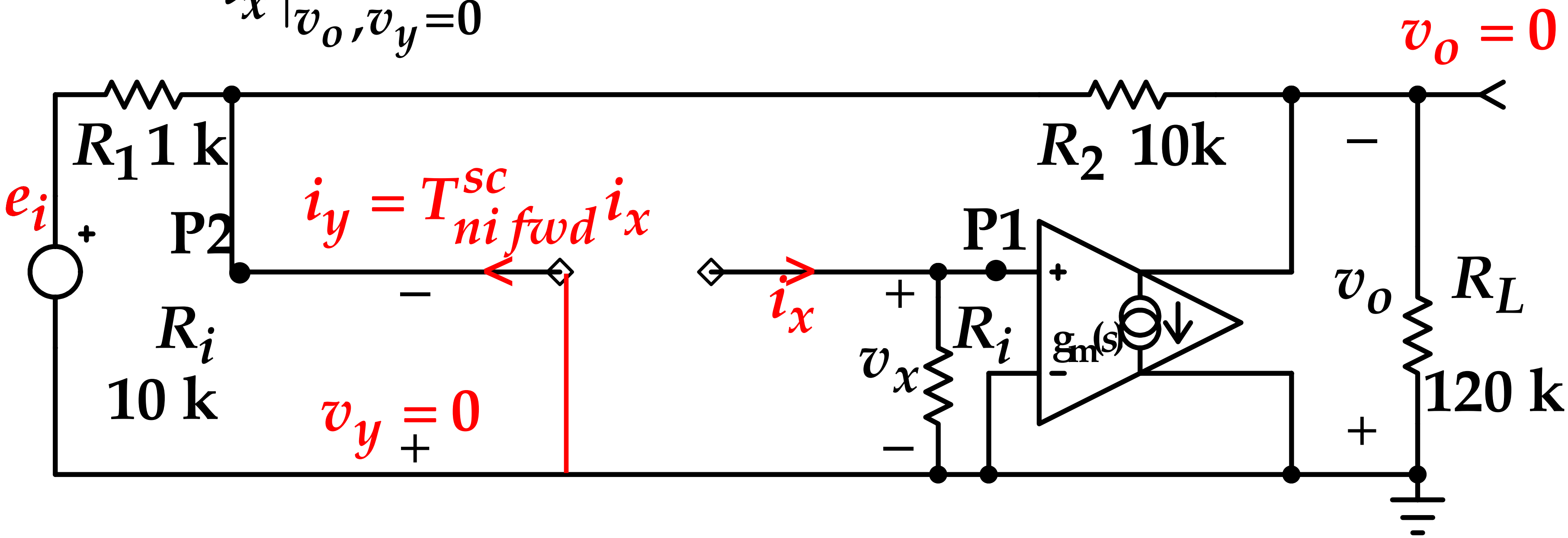
$$T = \frac{T_{fwd}}{1 + T_{rev}} \quad \textbf{where} \quad T_{rev} = T_{i\,rev}^{sc} \Big\| T_{v\,rev}^{oc} = 0,$$

so $T = T_{fwd} = T_{i\,fwd}^{sc} \Big\| T_{v\,fwd}^{oc}$

$$T(0) = g_m(0)\frac{R_L}{R_L + R_2 + R_1 \| R_i}\left(R_1 \| R_i\right) = 92 \Rightarrow 39.2\ \text{dB}$$

which is the same as $T_v(0)$ obtained from single voltage injection at the ideal voltage injection point P1.

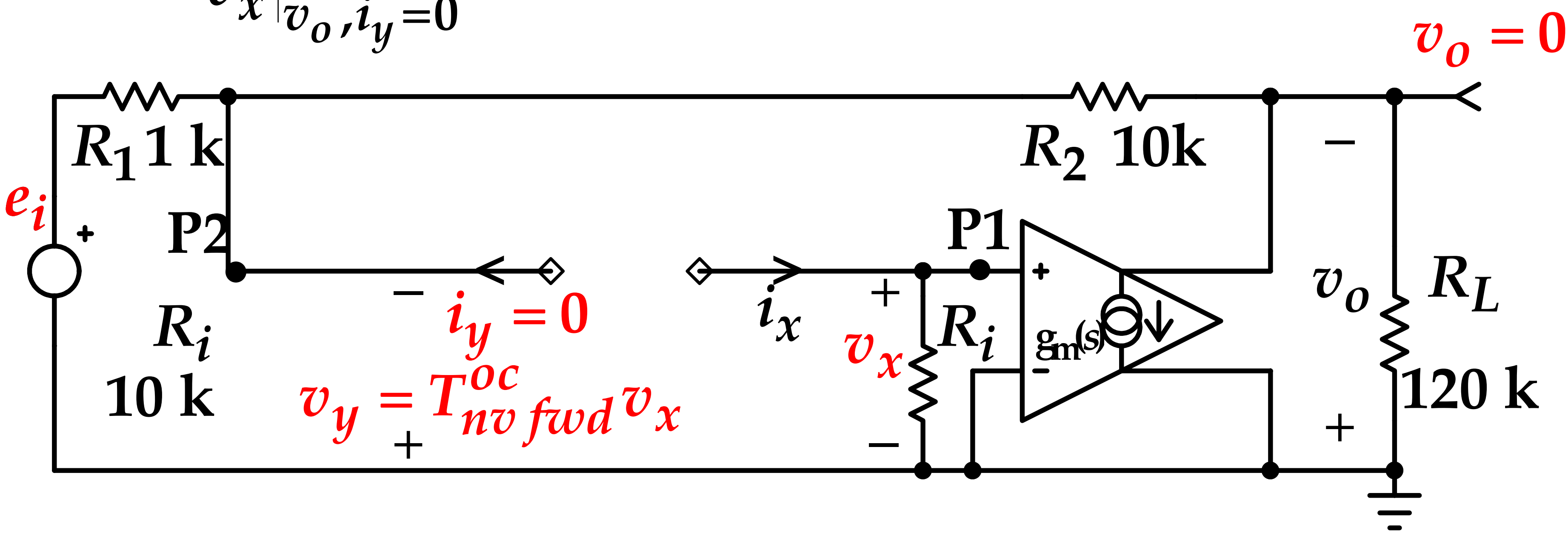
$$T_{ni}^{v_y} = \left.\frac{i_y}{i_x}\right|_{v_o, v_y = 0} \equiv T_{ni\,fwd}^{sc}$$



Because v_y and v_o are both nulled, there is no current in R_2 or R_L, so $i_x = 0$ and

$$T_{ni\,fwd}^{sc} = \infty$$

$$T_{nv}^{i_y} = \left.\frac{v_y}{v_x}\right|_{v_o, i_y = 0} \equiv T_{nv\,fwd}^{oc}$$



$$T_{nv\,fwd}^{oc} = g_m(s) R_2$$

$T_{ni\,rev}^{sc}$ and $T_{nv\,rev}^{oc}$ are both zero for the same reason that $T_{i\,rev}^{sc}$ and $T_{v\,rev}^{oc}$ are zero: there is no reverse current or voltage loop gain.

Finally,

$$T_n = \frac{T_{n\,fwd}}{1 + T_{n\,rev}} \text{ where } \quad T_{n\,rev} = T^{sc}_{ni\,rev} \| T^{oc}_{nv\,rev} = 0$$

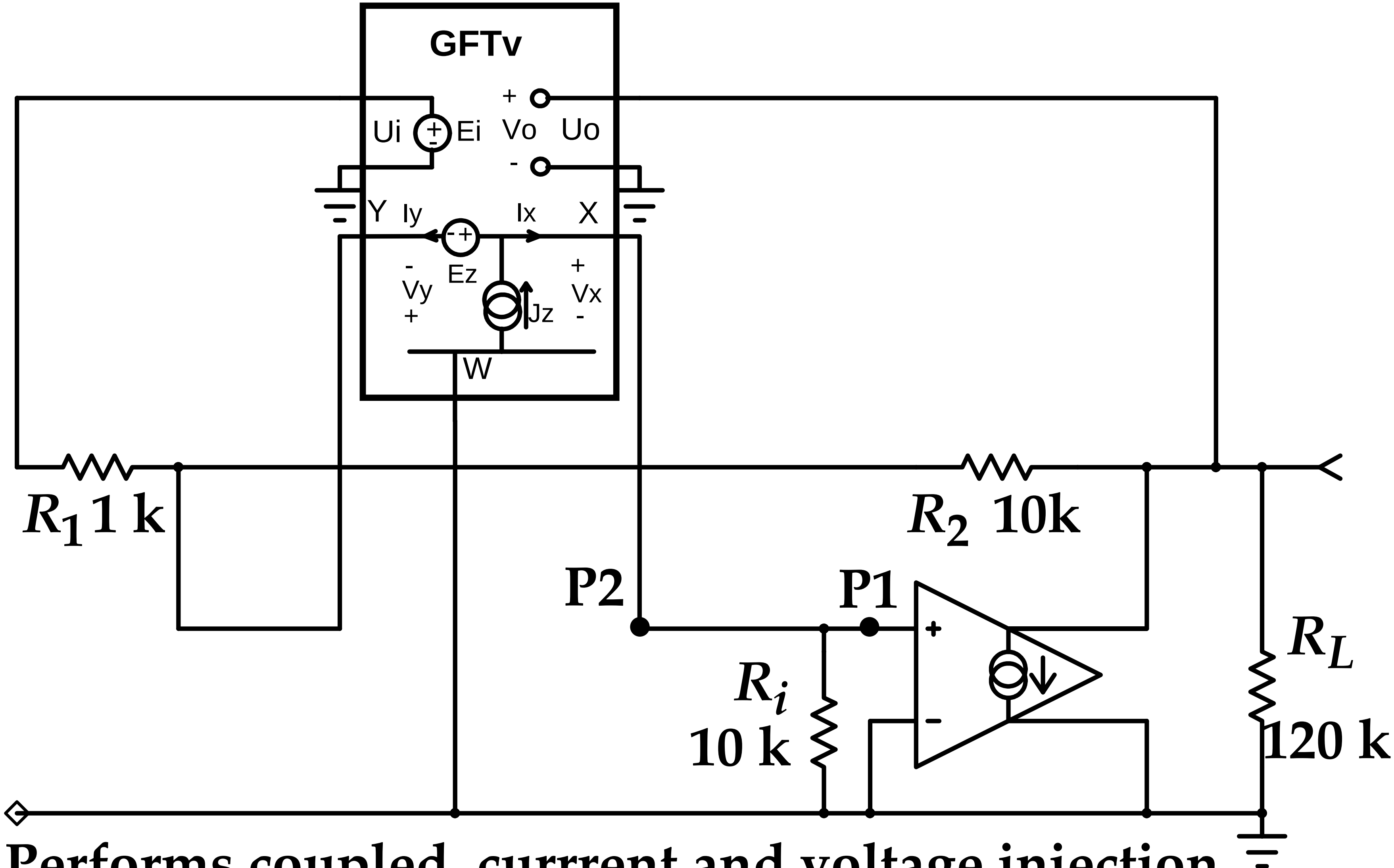
so $T_n = T_{n\,fwd} = T^{sc}_{ni\,fwd} \| T^{oc}_{nv\,fwd}$

However, $T^{sc}_{ni\,fwd} = \infty$, **so** $T_n = T^{oc}_{nv\,fwd}$

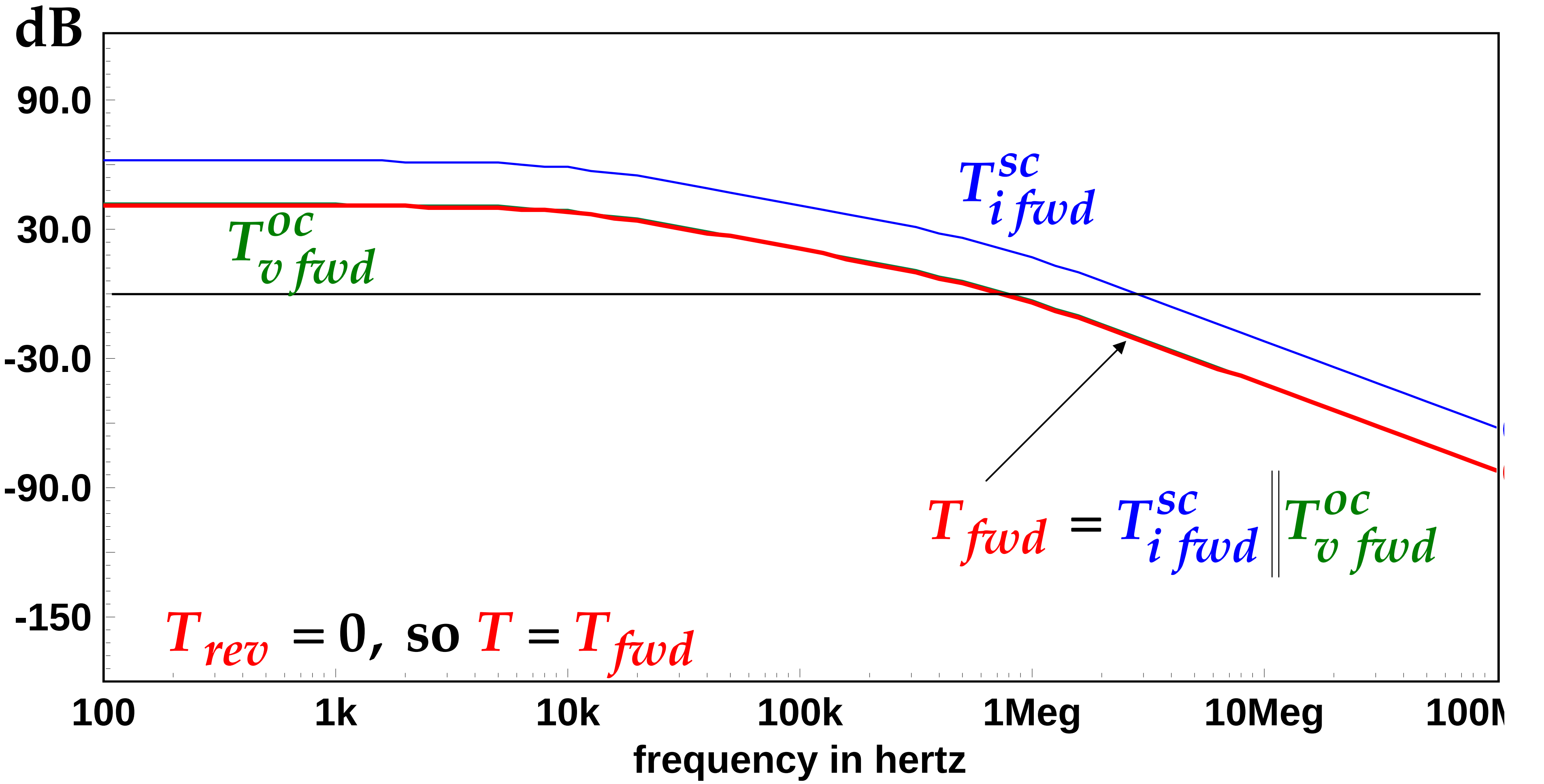
$$T_n = g_m(s)R_2$$

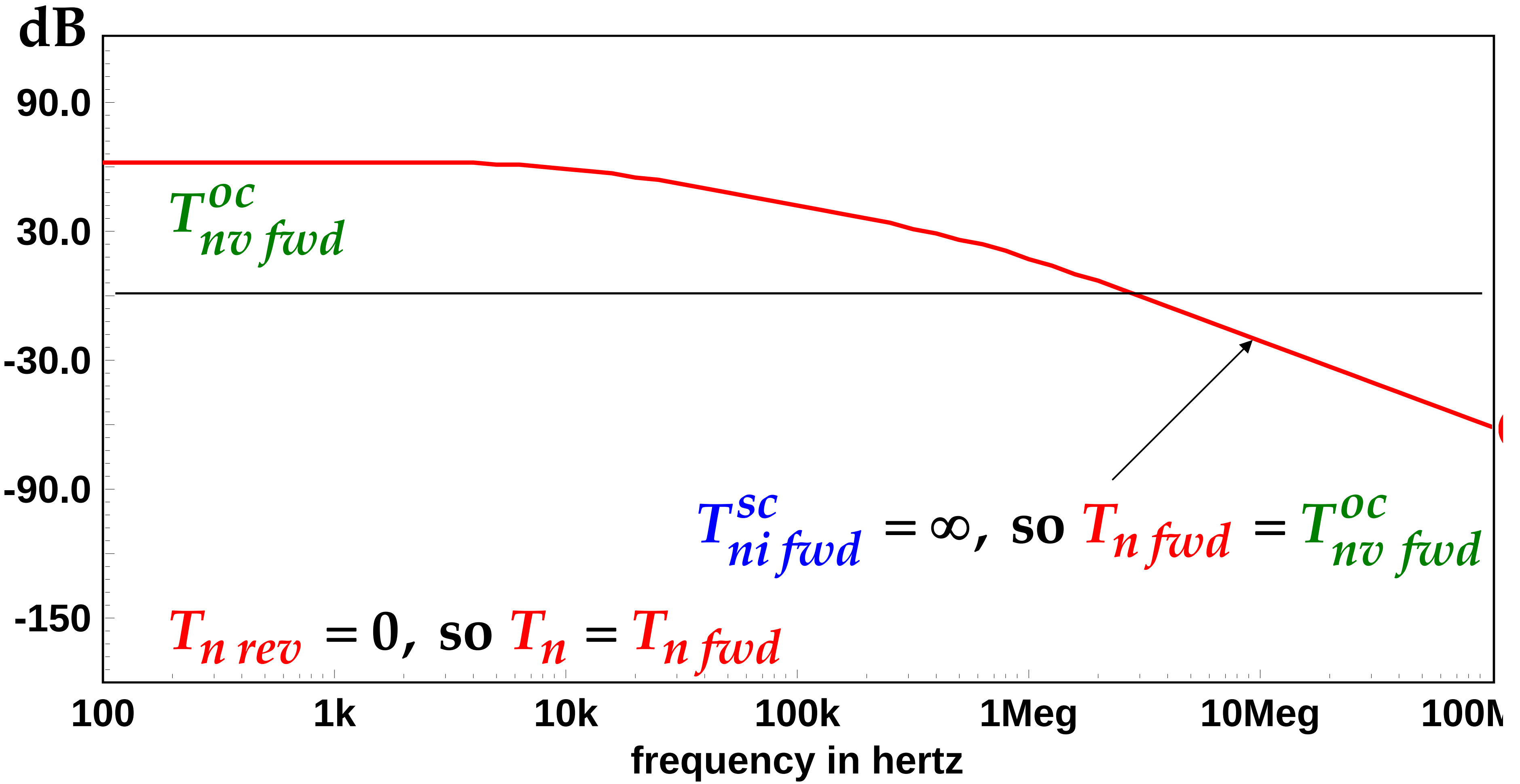
which is the same as T_{nv} **obtained from single voltage injection at the ideal voltage injection point P1.**

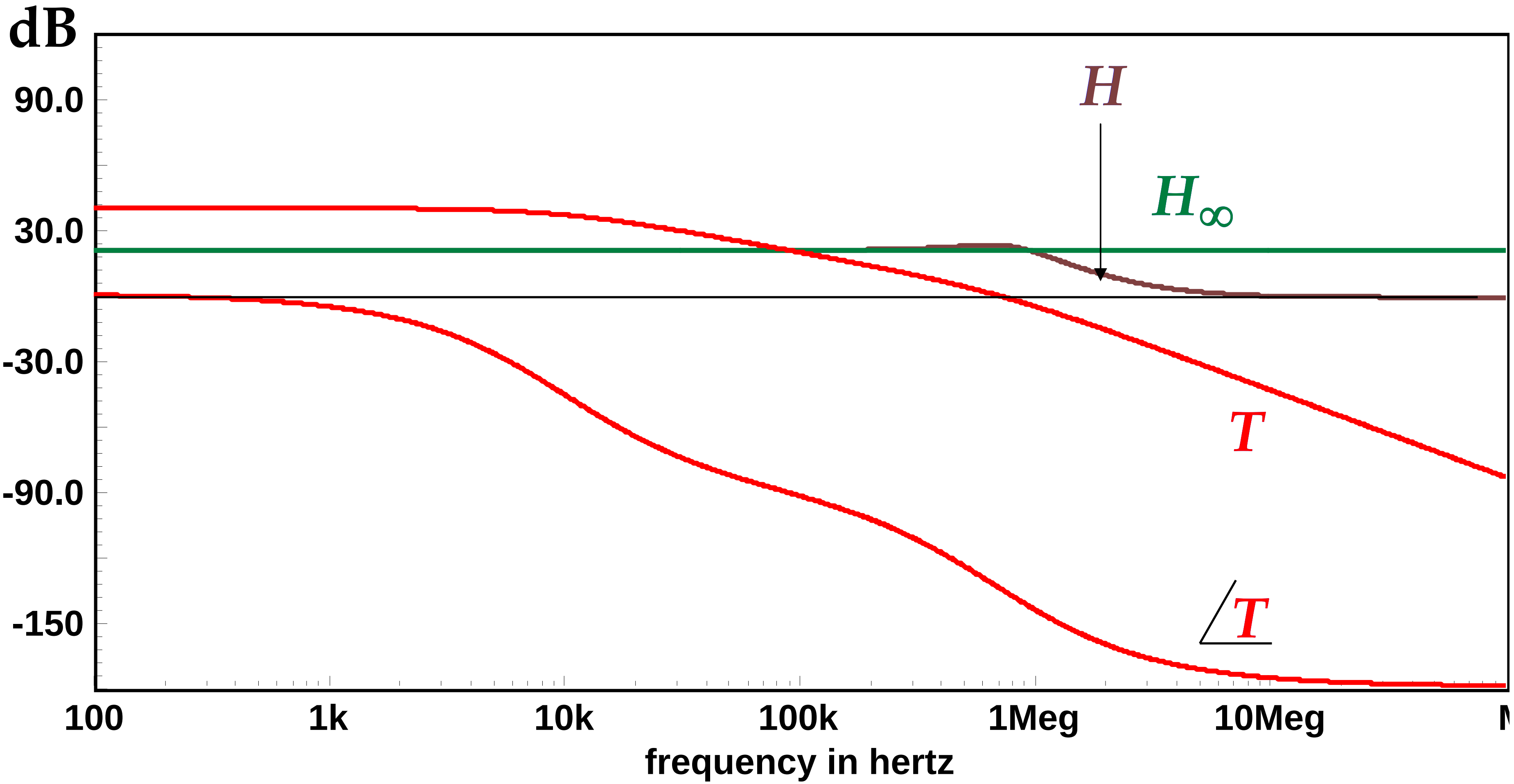
Intusoft ICAP/4 Circuit Simulator with GFT Template



Performs coupled currrent and voltage injection at the same nonideal point P2


dB
90.0
30.0
-30.0
-90.0
-150
$T^{sc}_{i\,fwd}$
$T^{oc}_{v\,fwd}$
$T_{fwd} = T^{sc}_{i\,fwd} \| T^{oc}_{v\,fwd}$
$T_{rev} = 0$, so $T = T_{fwd}$
100
1k
10k
100k
1Meg
10Meg
100M
frequency in hertz


dB
90.0
30.0
-30.0
-90.0
-150
$T^{oc}_{nv\,fwd}$
$T^{sc}_{ni\,fwd} = \infty$, so $T_{n\,fwd} = T^{oc}_{nv\,fwd}$
$T_{n\,rev} = 0$, so $T_n = T_{n\,fwd}$
100
1k
10k
100k
1Meg
10Meg
frequency in hertz



Results are the same as for single voltage injection at the ideal point P1

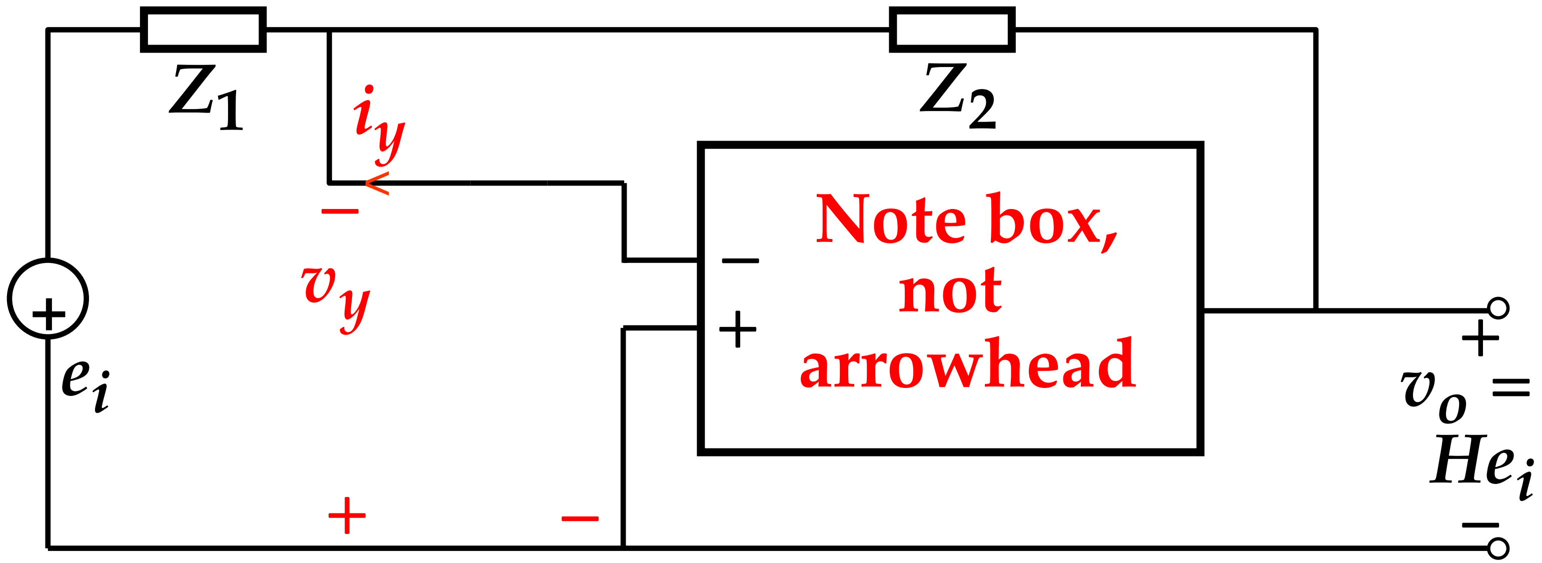
An *injection configuration* is defined by *which* test signals are adopted, and *where* they are injected.

Different injection configurations produce different sets of second-level TFs, but each set combines to give the *same* first-level TF H.

Therefore, the choice of Test Signal Injection Configuration is crucial in making the second-level TFs H_∞, T, T_n have the desired interpretations.

Basic Inverting Opamp

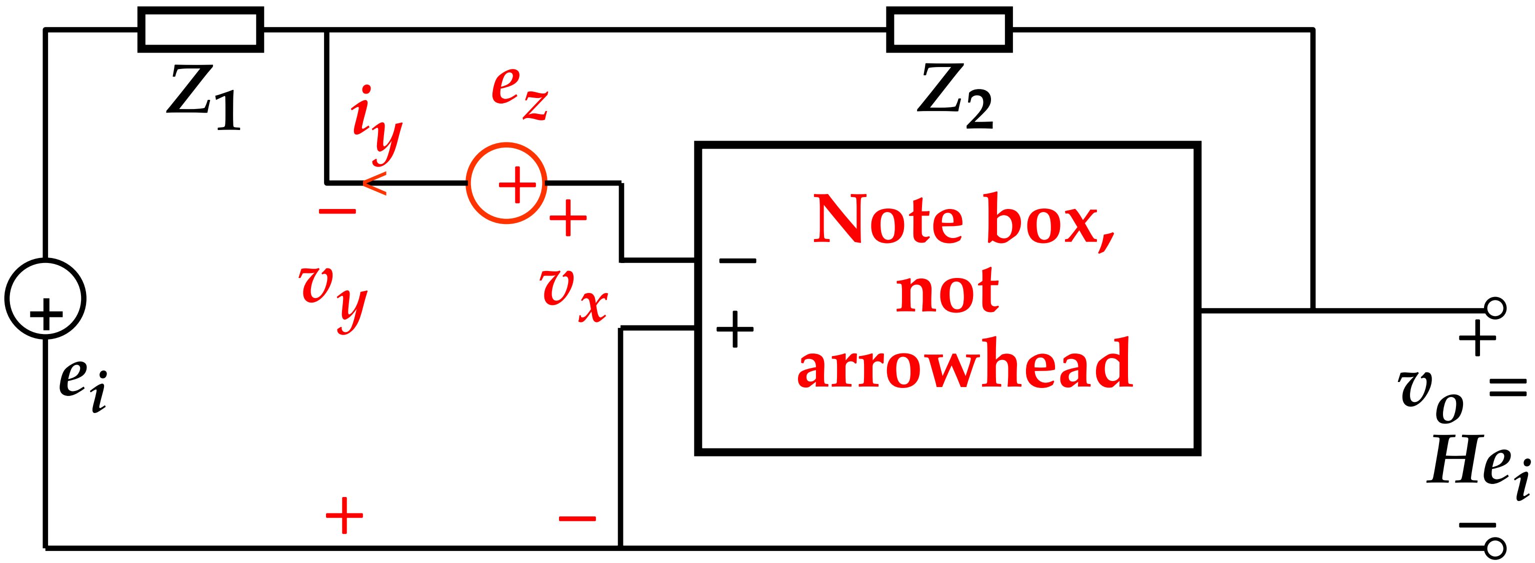
Identify error voltage v_y and error current i_y :



$$H_\infty \equiv \text{ideal closed-loop gain} = H^{i_y v_y} = \frac{Z_2}{Z_1}$$

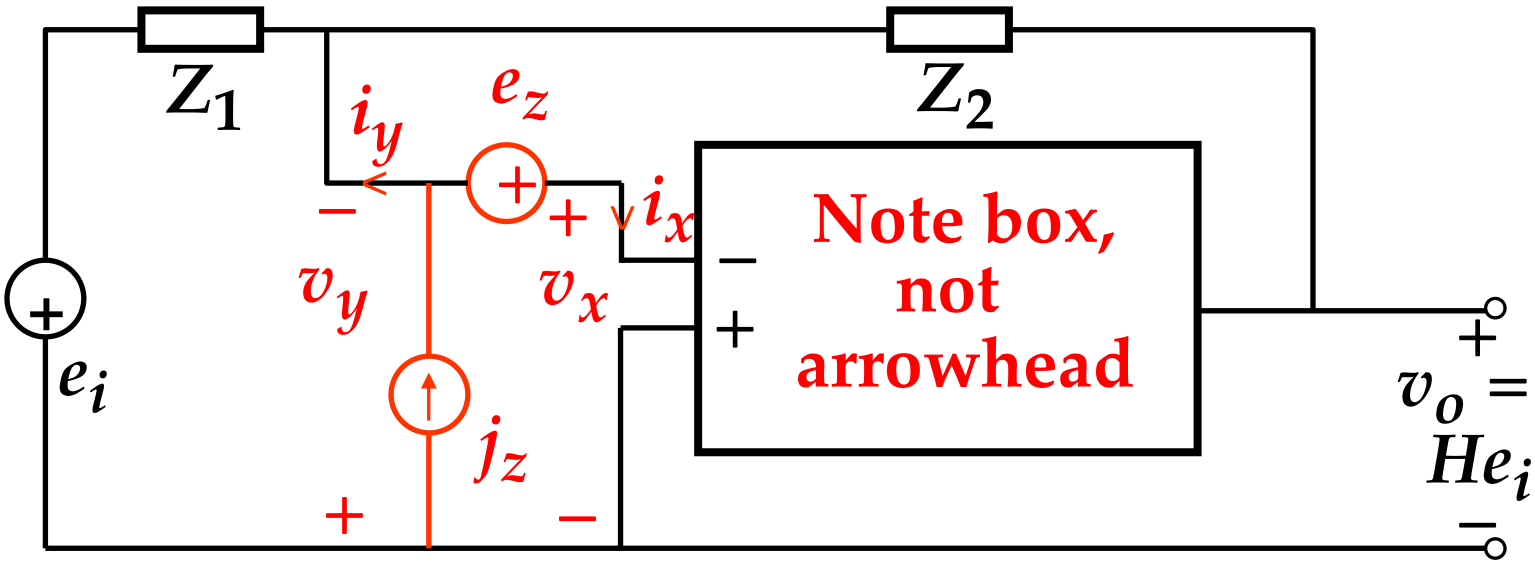
(Requirement 1)

Identify v_x drive to the forward path, and hence where e_z is to be injected (Requirement 2):



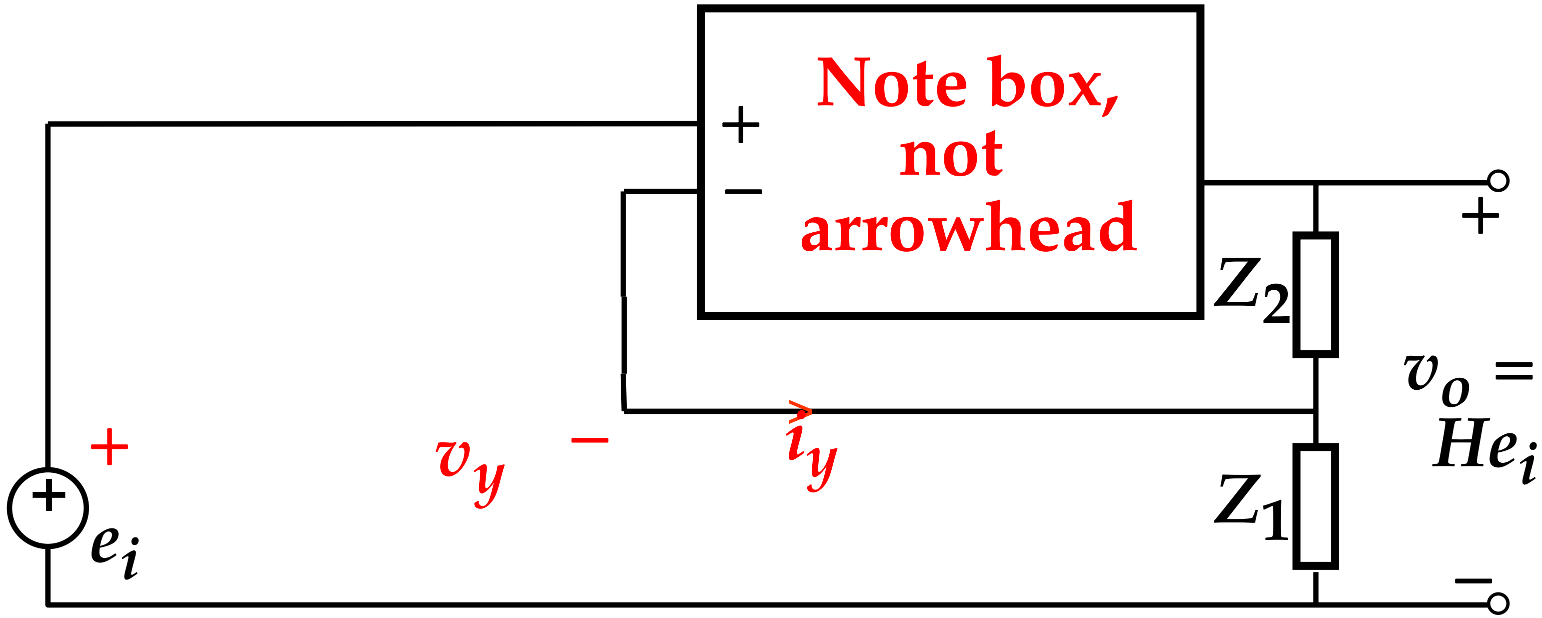
When v_y is nulled, is i_y automatically nulled?

If not, identify i_x drive to the forward path, and hence where j_z is to be injected:



Basic Noninverting Opamp

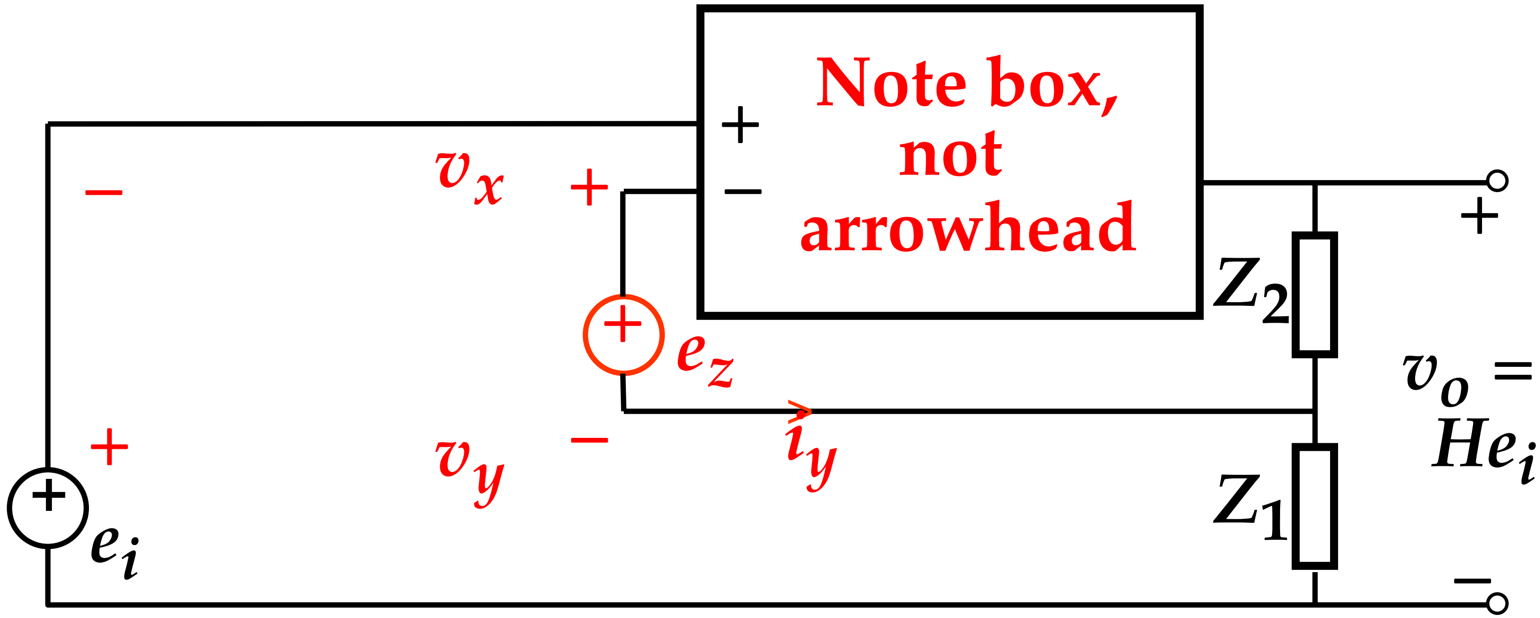
Identify error voltage v_y and error current i_y :



$$H_\infty \equiv \text{ideal closed-loop gain} = H^{i_y v_y} = \frac{Z_1 + Z_2}{Z_1}$$

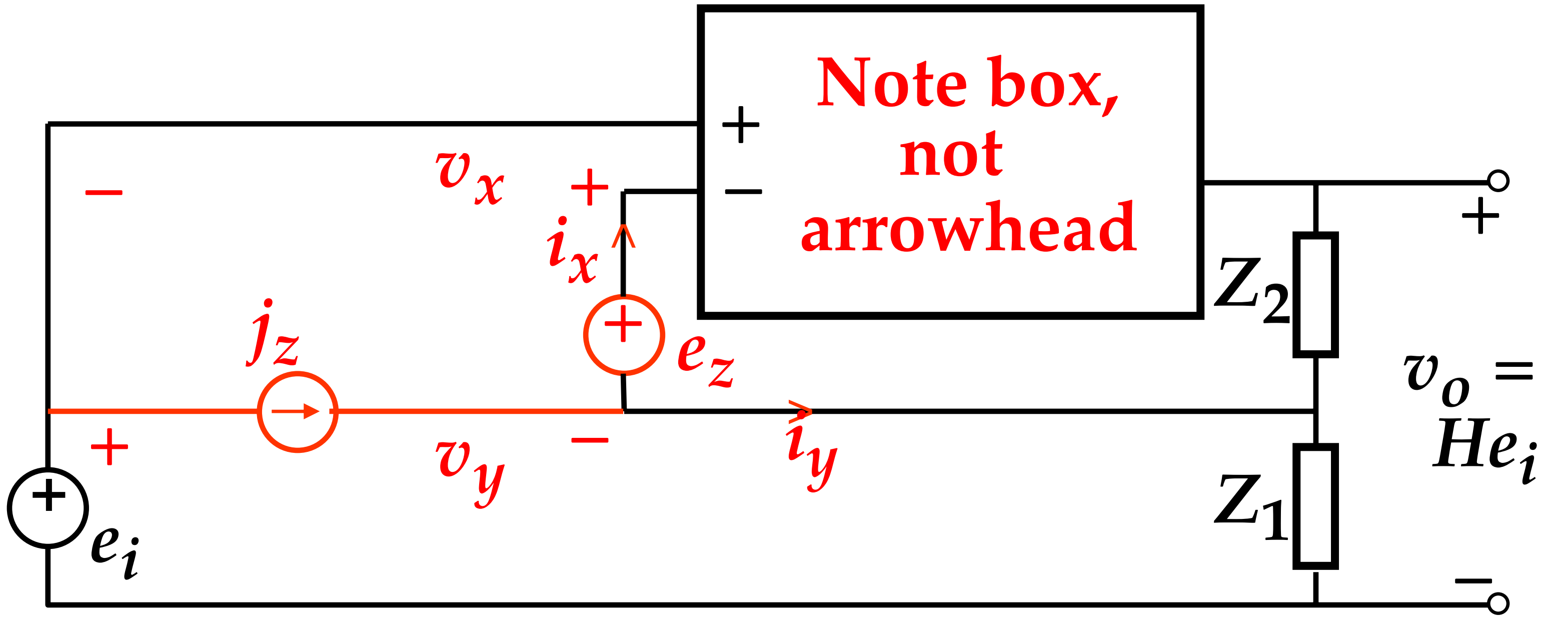
(Requirement 1)

Identify v_x drive to the forward path, and hence where e_z is to be injected (Requirement 2):

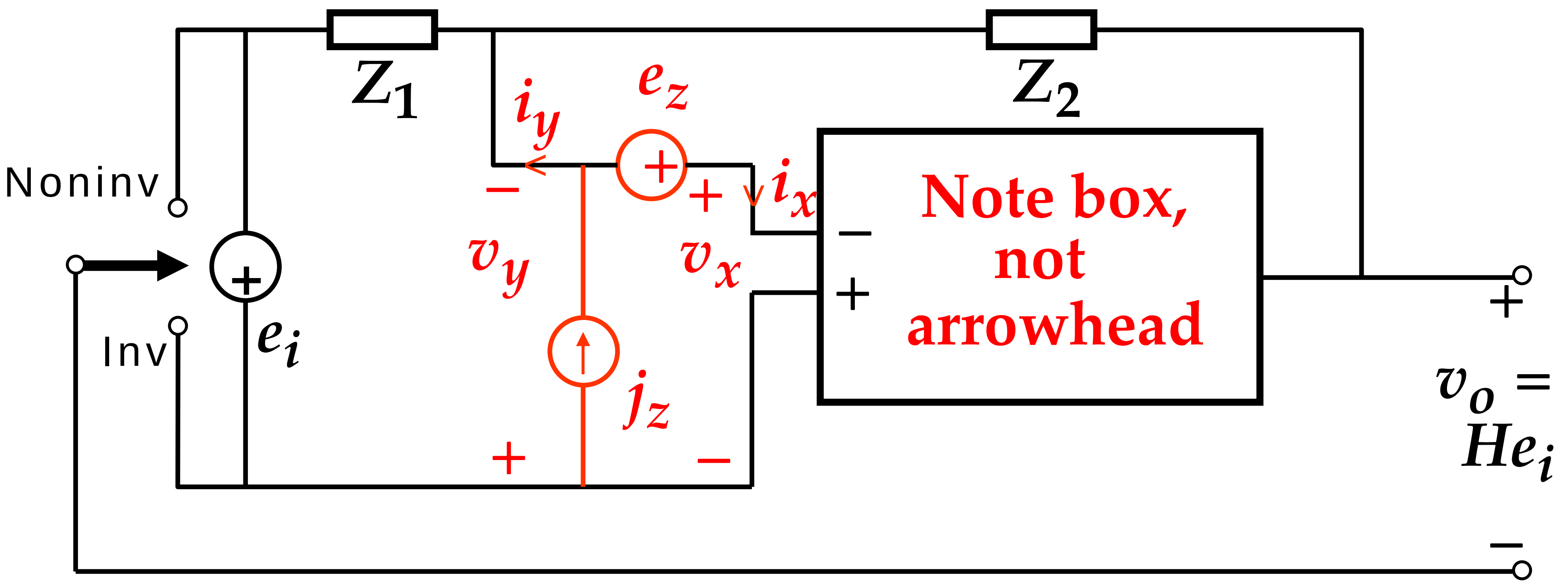


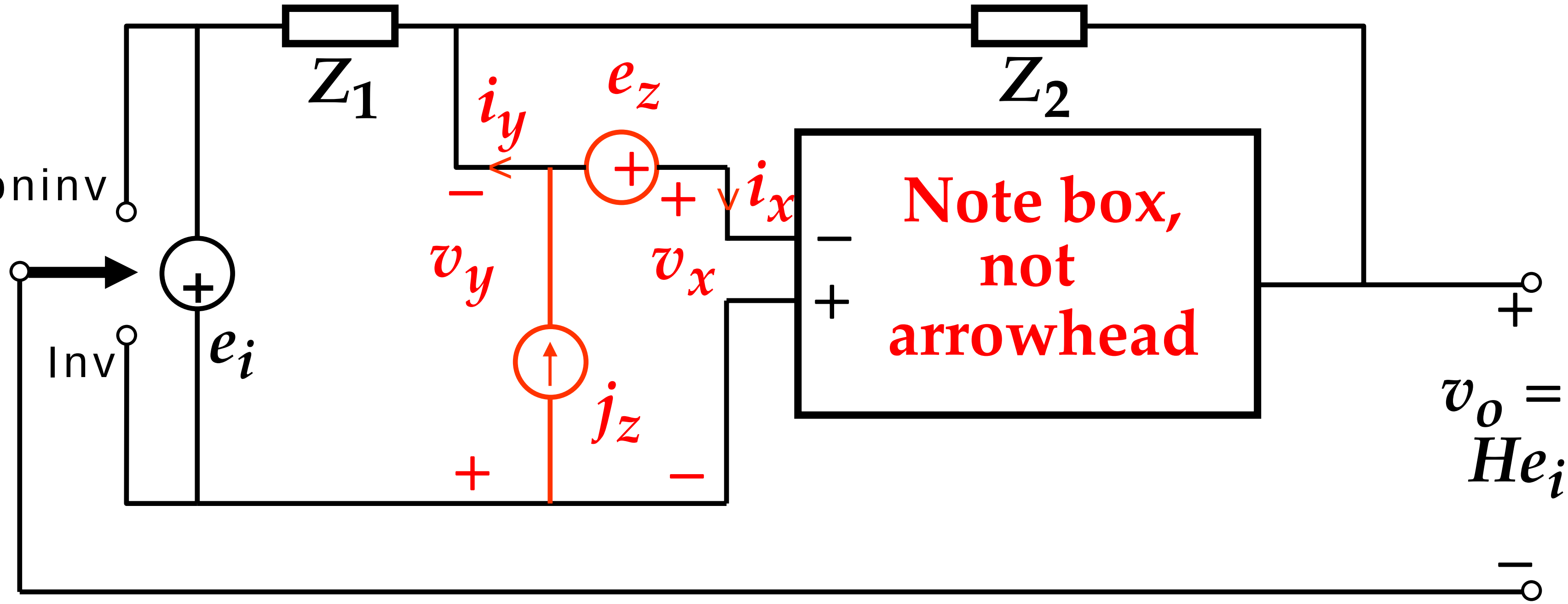
When v_y is nulled, is i_y automatically nulled?

If not, identify i_x drive to the forward path, and hence where j_z is to be injected:



Despite the apparently different layouts, both the Inv and Noninv configurations are the same, except that the reference node for the output voltage is different:





$$H_{\text{Inv}} = H_{\infty\text{Inv}} \frac{1+\frac{1}{T_{n\text{Inv}}}}{1+\frac{1}{T}} \qquad H_{\text{Noninv}} = H_{\infty\text{Noninv}} \frac{1+\frac{1}{T_{n\text{Noninv}}}}{1+\frac{1}{T}}$$

The principal loop gain T is the *same* for both;

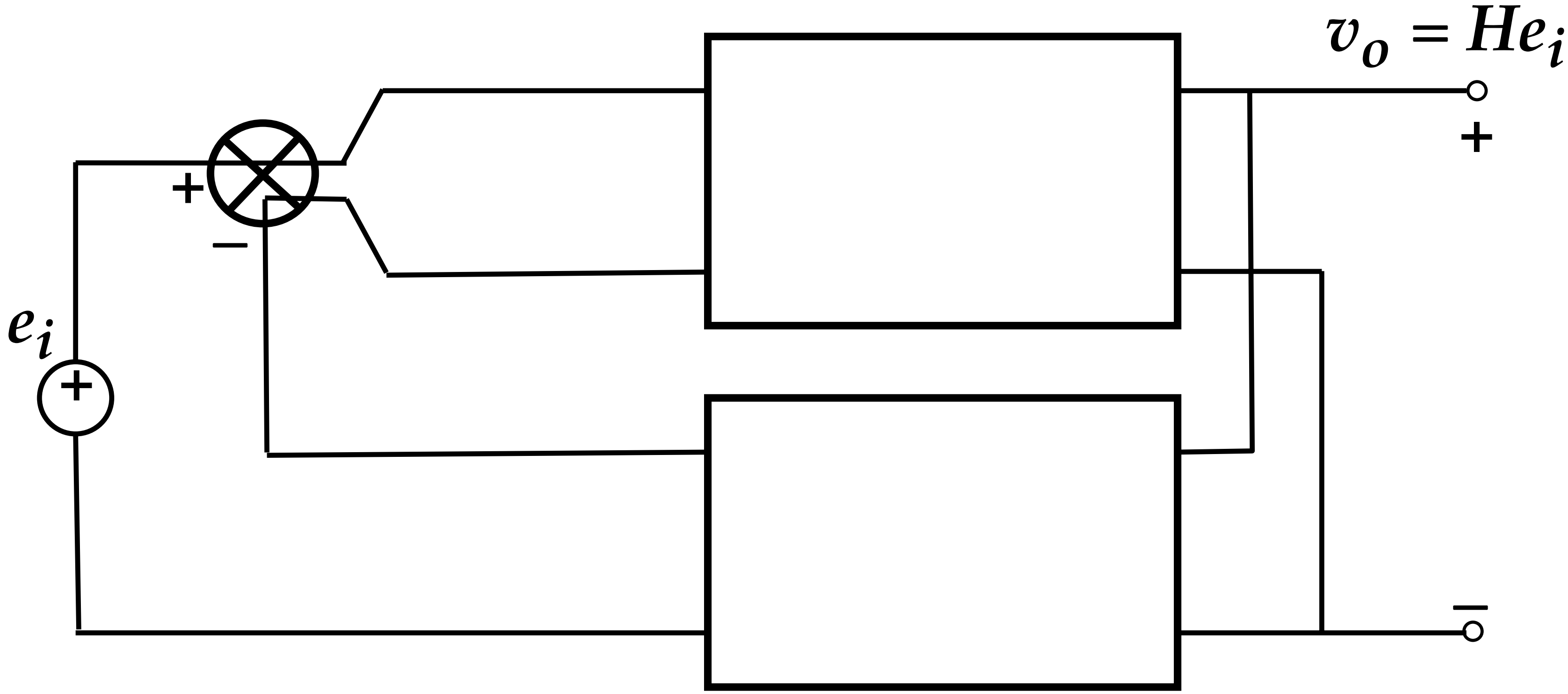
The amplifier CM gain affects $T_{n\text{Noninv}}$, but does not affect $H_{\infty\text{Noninv}}$, $H_{\infty\text{Inv}}$, $T_{n\text{Inv}}$, or T.

SUMMARY

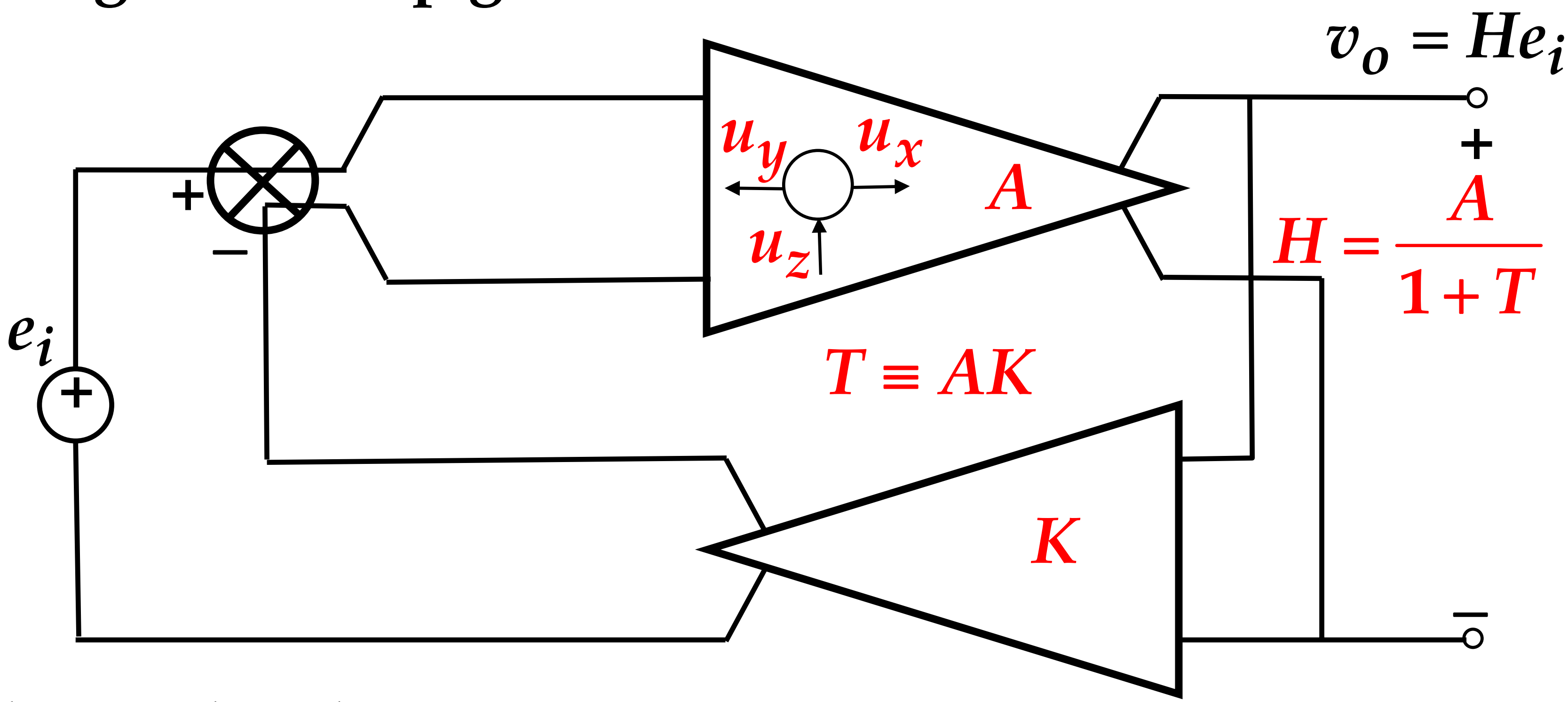
History of the conventional approach

The GFT: The Final Solution approach

Analysis History: Replace the boxes with arrowheads:



Spotlight the loop gain:

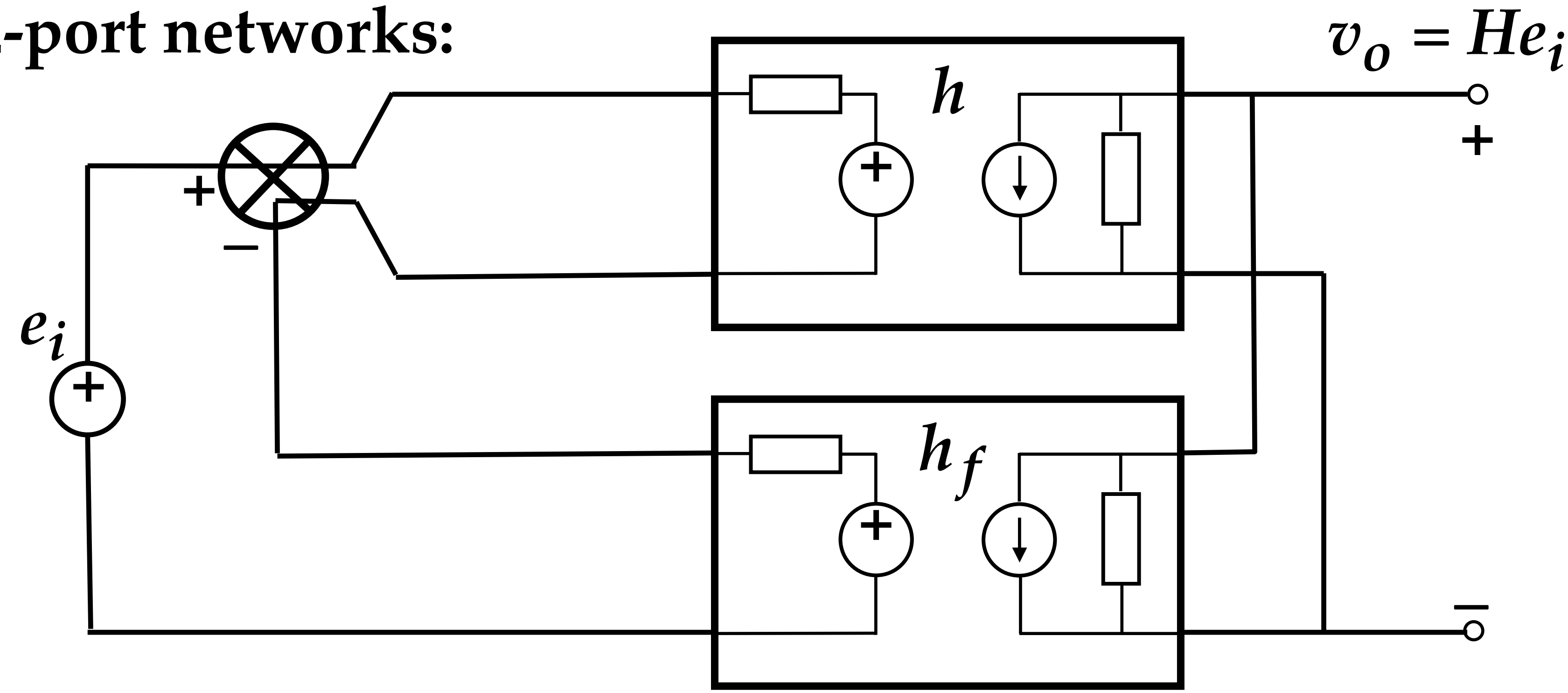


Development chronology:

1. Disconnect the feedback path, calculate *A* and *K* separately: ignores loading
2. Inject test signal where there is no loading (ideal injection point)
3. Inject coupled test signals (nonideal injection point)

Result: reverse transmission is ignored

Alternative: set up the forward and feedback paths as 2-port networks:



This requires four different sets of 2-port parameters for the four feedback connections, and it's still not clear how to account for reverse loop gain.

Disadvantage:

The model itself is inaccurate, in three of the four connections, because common-mode gain is ignored.

Greater Disadvantage:

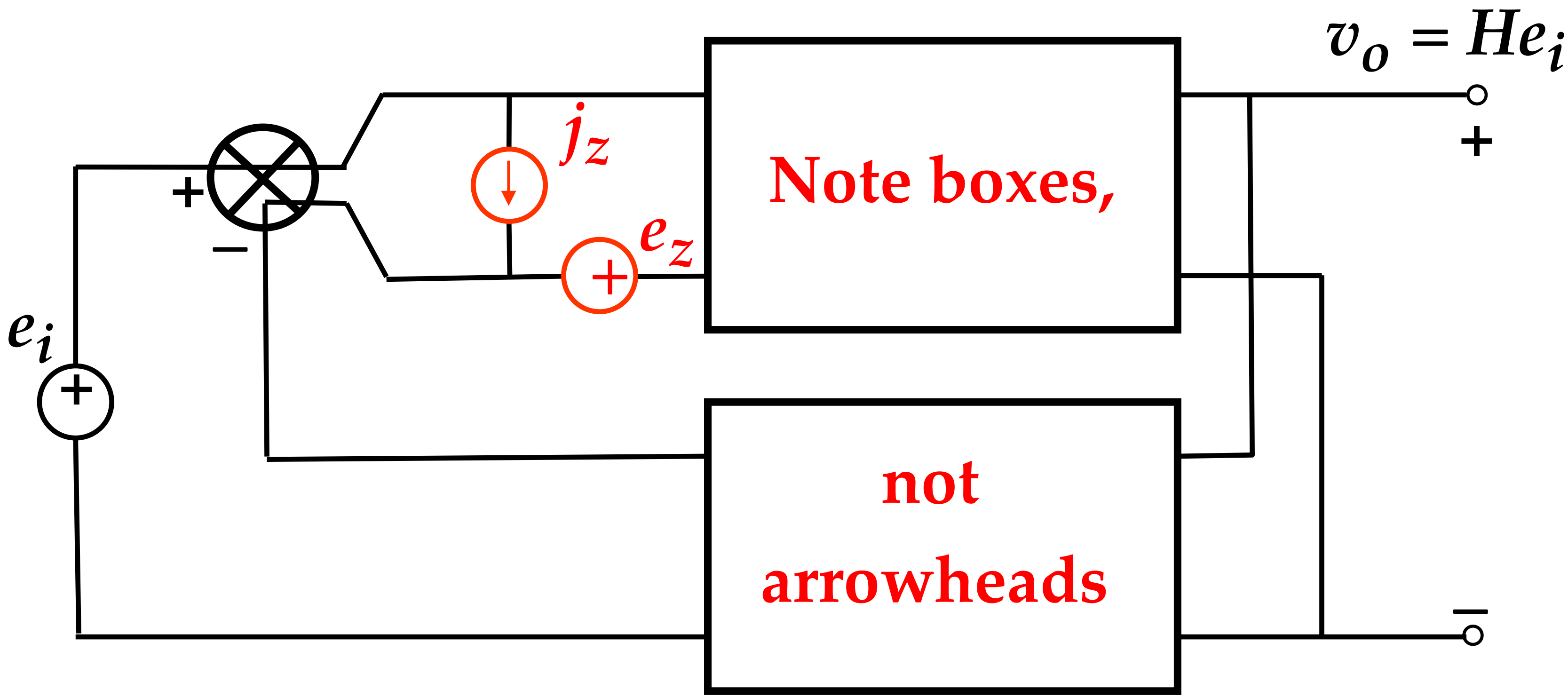
Essentially useless for design, because the circuit elements are buried inside the 2-port parameters, which are themselves buried in expressions for closed-loop gain and loop gain.

WHAT THE GFT DOES:

Defines and calculates the Principal Loop Gain

Identifies and calculates nonidealities

The GFT Approach: the Final Solution



Inject coupled test signals at error summing point.

The GFT gives all the second-level TFs directly

in terms of the circuit elements.

HOW THE GFT DOES IT:

$$H = H_\infty \frac{1+\frac{1}{T_n}}{1+\frac{1}{T}}$$

The Test Signal Injection Configuration is chosen to meet the following criteria:

1. The test signal(s) must be injected at the error summing point. This makes H_∞ equal to the Ideal Closed-Loop Gain $1/K$, the reciprocal of the feedback ratio.

2. The test signal(s) must be injected inside the major loop, but outside any minor loops. This makes T represent the Principal Loop Gain.

1 plus 2 makes the Null Loop Gain T_n represent all the nonidealities, including:

forward transmission through the feedback path

reverse transmission through the forward path

If T_n>>1, the nonidealities are negligible.

Injection of both voltage and current test signals at a nonideal point includes, as special cases, injection of only one test signal at an ideal point, so you don't have to know in advance whether or not the injection point is ideal.

Benefit:

Results include reverse transmission in both forward and feedback paths, and thus include reverse loop gain. Approximations are easily made, if desired.

Benefit:

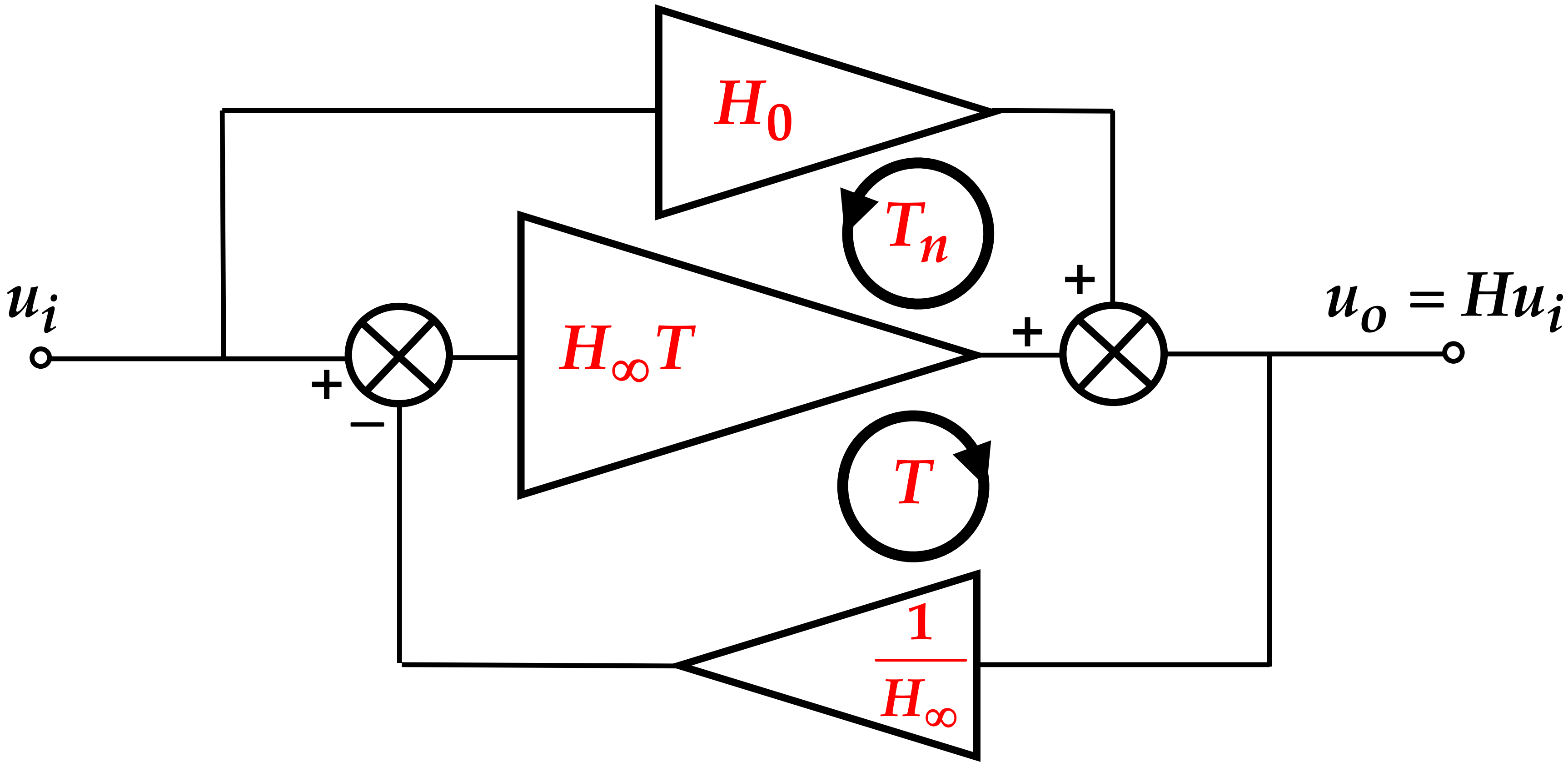
All four second level TFs H_∞, T, T_n, H_0 are produced, either numerically by Intusoft ICAP/4 GFT Template, or symbolically by ndi and dnti analysis.

Benefit:

The second-level TFs combine in the GFT to produce the first-level result, the closed-loop TF H:

$$H = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}$$

This result fits the block diagram:



Greater Benefit:

Thus, the block diagram, with unidirectional blocks, is a *result* of simple and straightforward *exact* calculation, *not an initial assumption.*

Greatest Benefit:

The second-level TFs H_∞, T, T_n, H_0 are obtained directly in terms of the circuit elements, which makes it possible for the results to be used backwards for design. This is the principal objective of

Design-Oriented Analysis:

the Only Kind of Analysis Worth Doing.